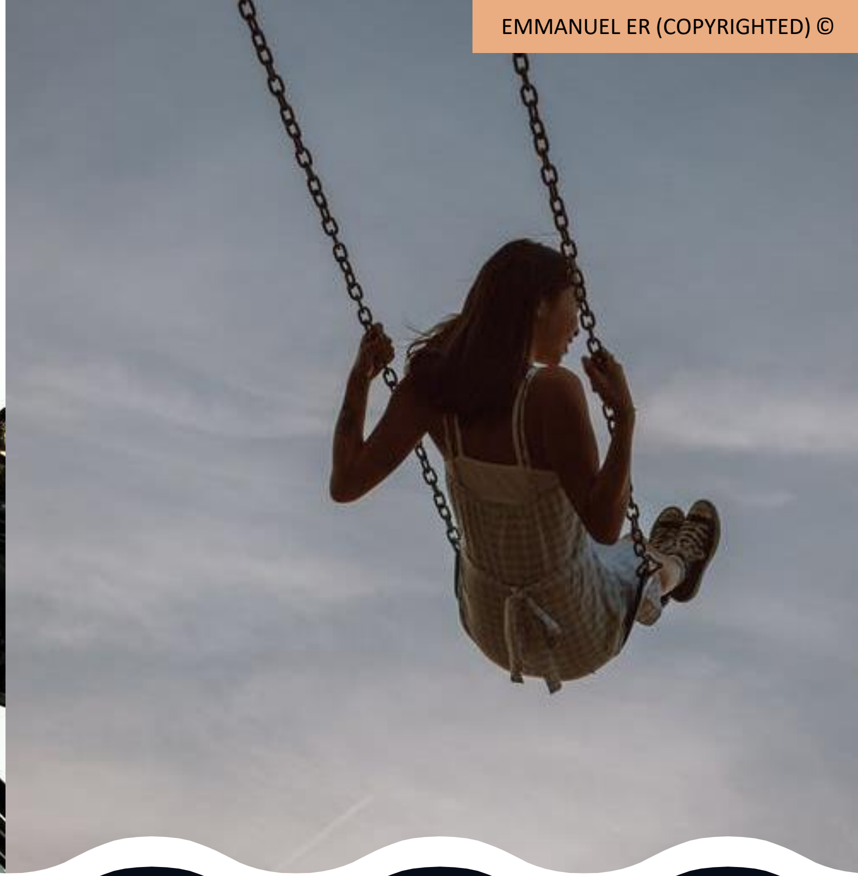




OSCILLATIONS

Overmugged

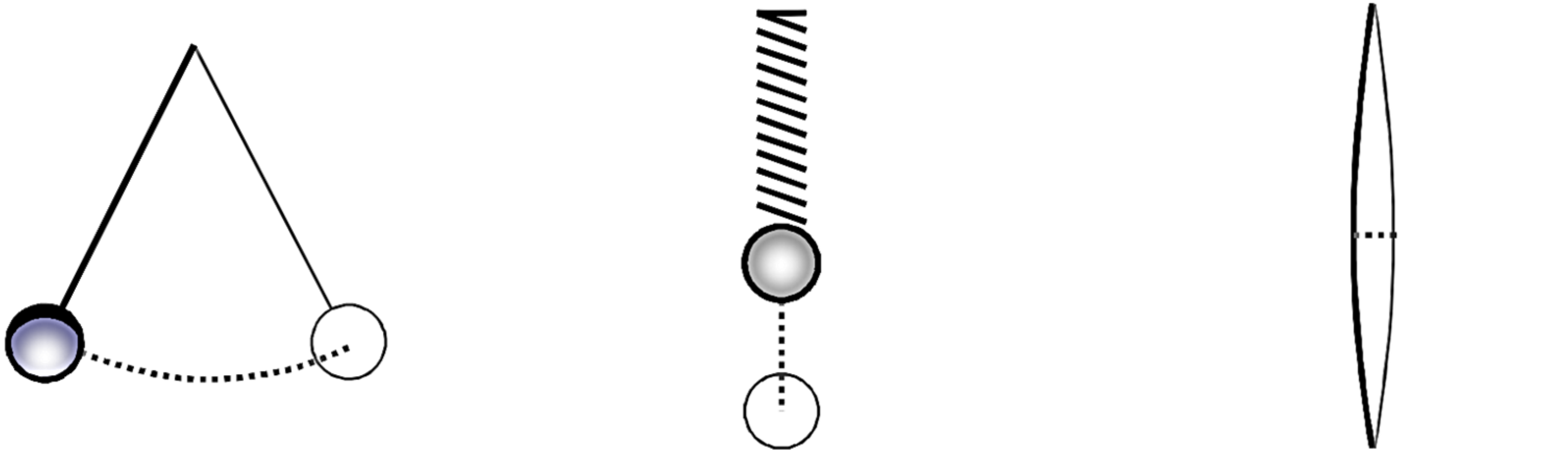
CONTENTS

- Oscillations
- Simple harmonic motion
- Energy in simple harmonic motion
- Damped and forced oscillations: resonance

Oscillations

- refer to the repetitive back and forth motion of an object along the same path, with the motion repeating itself in equal time intervals

Examples



(a) swinging pendulum　(b) vibrating mass-spring system　(c) vibrating string

FREE, DAMPED, and FORCED Oscillation

FREE

- Occur when a system oscillates with a natural frequency under a **restoring force.**
- Energy is conserved and quantities such as amplitude and period remain the same.

DAMPED

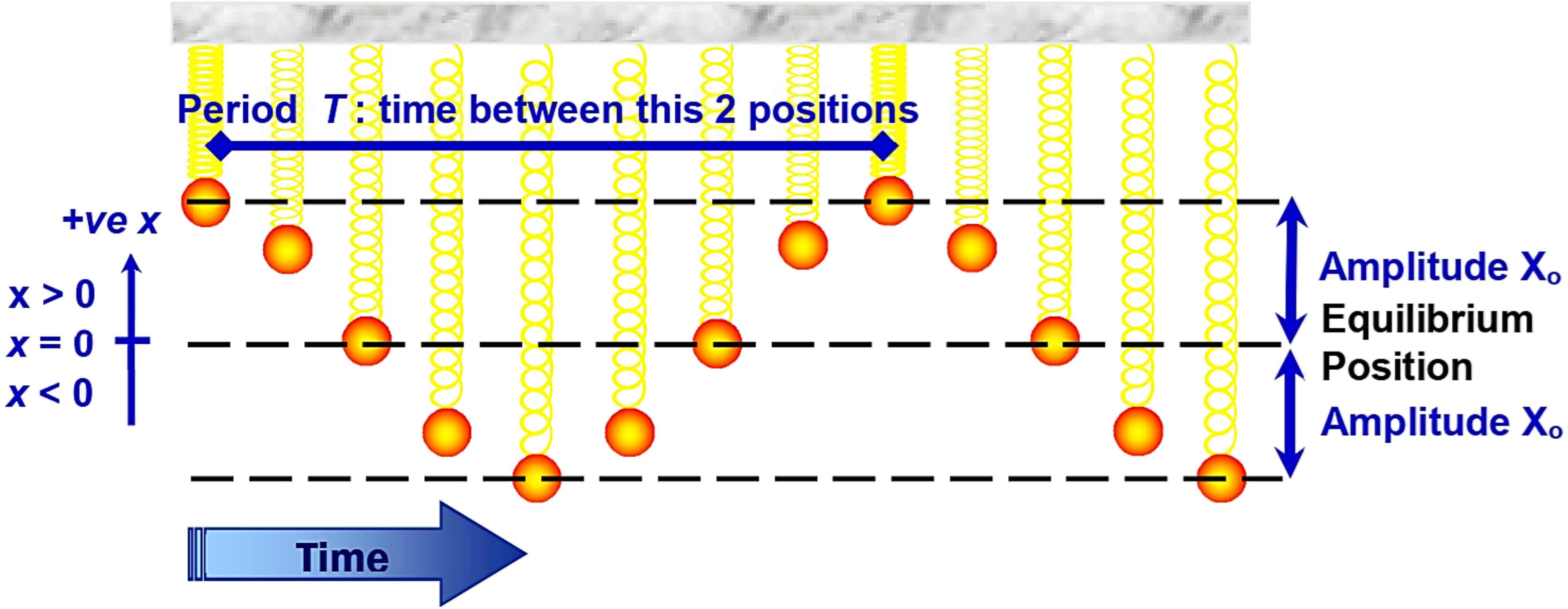
- Occur when the system experiences resistive forces.
- Oscillation dies out over time

FORCED

- Caused by continual input of energy source to an oscillating system to compensate the loss due to damping in order to maintain the amplitude of the oscillation.
- The external force is the **driving force**.

Quantities of Oscillations

Consider displacement with time of a spring-mass oscillating in frictionless environment.



Quantities of Oscillations

Equilibrium or neutral position

- Is the position at which no net force acts on the oscillating mass

Displacement (x)

- Is the distance of the oscillating mass from its equilibrium position at any instant.

Amplitude (x_0)

- Is the maximum displacement of the oscillating mass from the equilibrium position in either direction

Period (*T*)

- Is the time taken for one complete oscillation of the oscillating mass

Frequency (*f*)

- Is the number of complete to-and-from oscillations [per unit time made by the oscillating mass.

$$f = \frac{1}{T}$$

Quantities of Oscillations

Phase (ϕ)

- Is an angular measure (in either degrees or radians) of the fraction of a cycle that has been completed by the oscillating mass.

Phase Difference ($\Delta\phi$)

- Is a measure of how much an oscillation is out of step with itself at 2 different instances in time (or how much 2 oscillations are out of step with each other at the same instant in time).

In phase: phase difference is zero
Out of phase: phase difference is not zero
Antiphase: phase difference is 180° or π rads

Angular frequency (ω)

- Is the rate of change of phase of oscillating mass

Simple Harmonic Motion

S.H.M. is a type of oscillatory motion whose acceleration is always

1. directly proportional to its displacement from the equilibrium position and
2. directed towards the equilibrium point (or is always directed opposite to of its displacement)

$$\vec{a} = -\omega^2 \vec{x}$$

$$v = \pm\omega\sqrt{x_0^2 - x^2}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

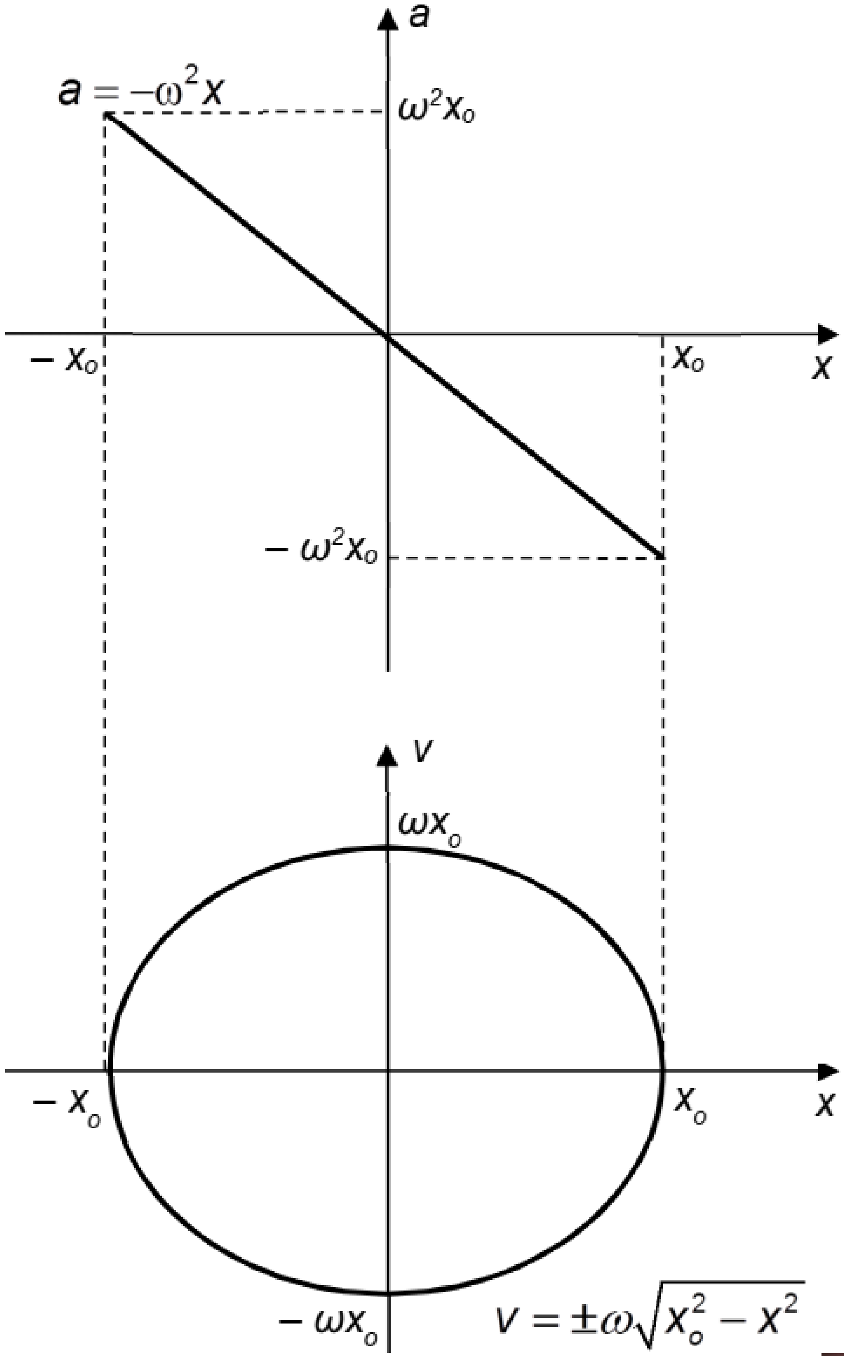
where $a =$ acceleration of oscillator at certain position
$x =$ displacement from equilibrium position
$v =$ velocity of the oscillator
$\omega =$ angular frequency of oscillator (constant)
$f =$ frequency, $T =$ period of oscillation

Simple Harmonic Motion

Graphical Representation

a_0: maximum acceleration
x_0: amplitude

gradient: $-\omega^2$



Feature of $a - x$ graph

- a straight line passing through origin → $a \propto x$
- negative gradient → *a* and *x* acts in opposite direction

1. Angular frequency and frequency are NOT the same.
2. The symbol and equations for angular frequency and angular velocity are the same but they are different quantities.

Practice Example 1

A spring-mass system is vertically suspended from a ceiling. The acceleration of the block of mass m is given by

$$a = -\frac{k}{m}x$$

where x is the displacement of the block. Discuss why the block's motion is simple harmonic.

Practice Example 2

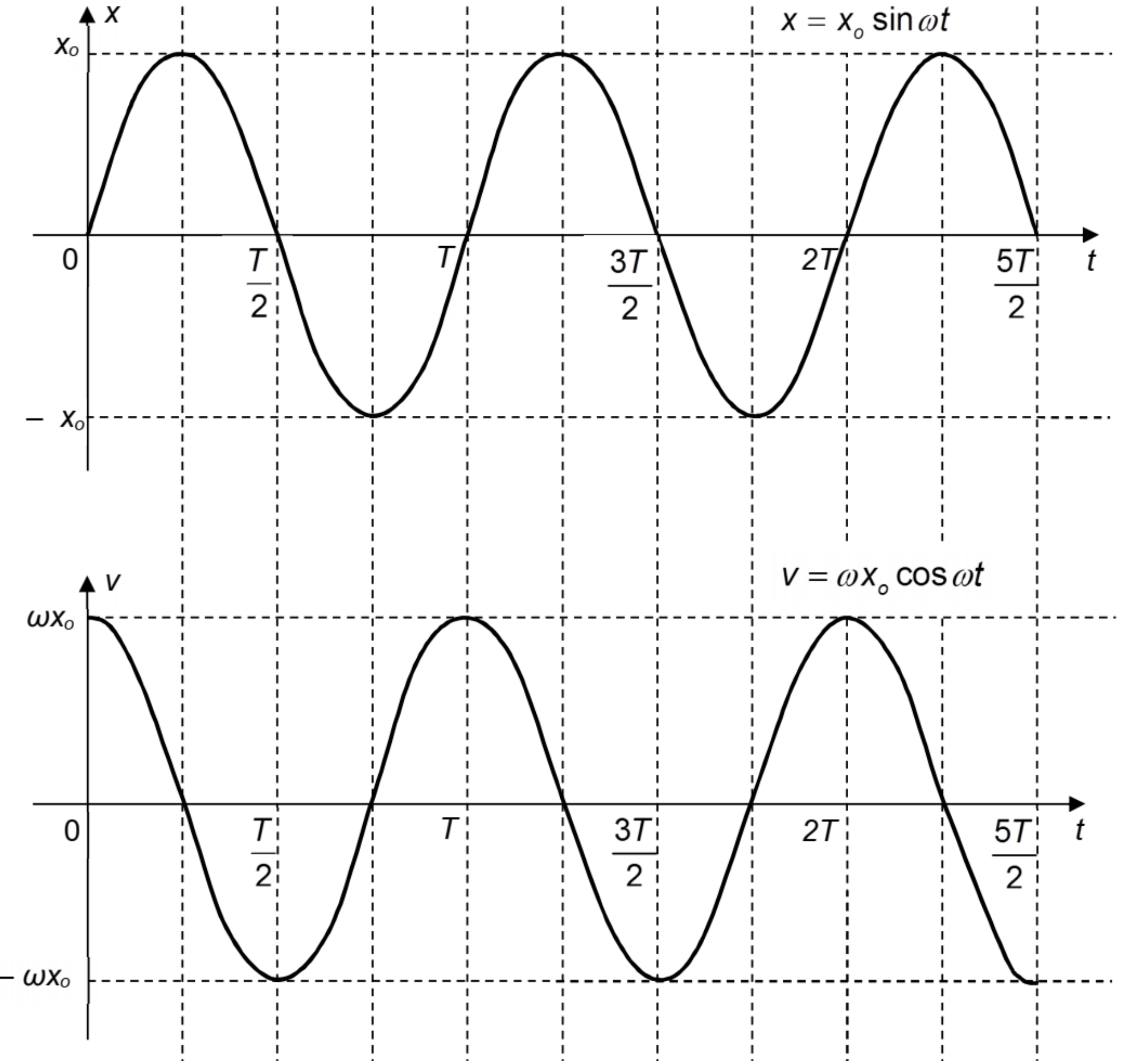
Consider two identical pendulums. One takes 26.0 seconds to complete 15 oscillations while the other one lags 1.23 seconds behind the first. Determine the phase difference between the pendulums.

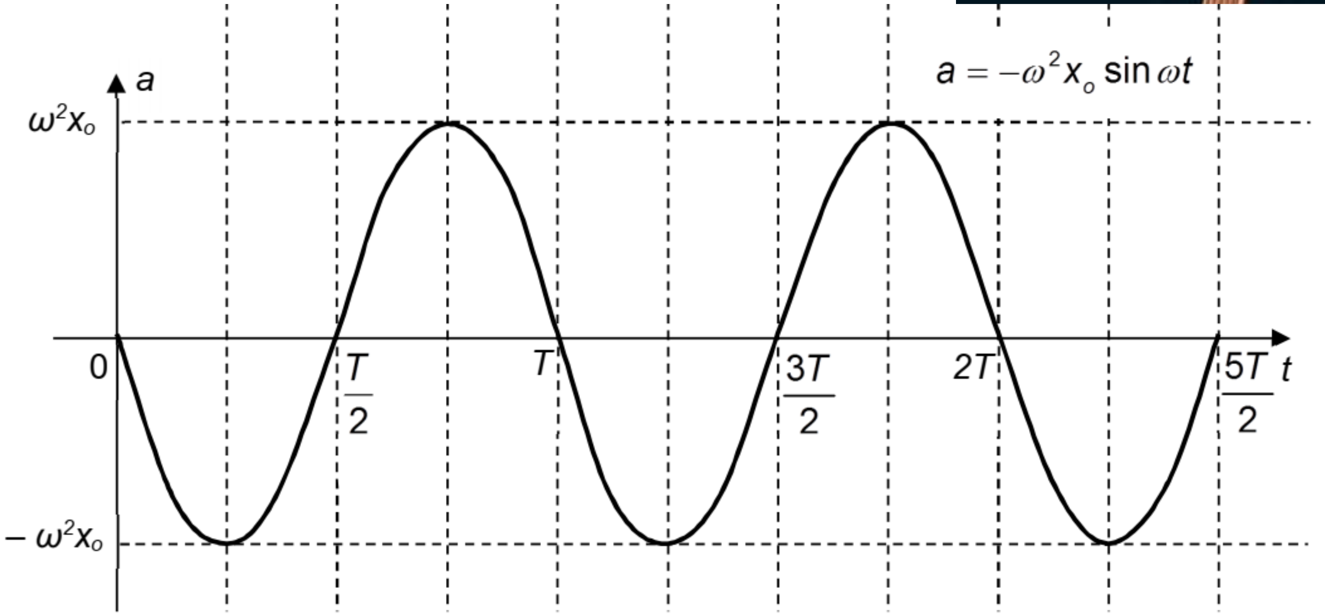
Kinematics of S.H.M.

The kinematic relationship of a S.H.M. can be summarized as follows:

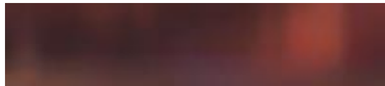
	Variation with *x*	Variation with t	
		Starting at $x = 0$ when $t = 0$	Starting at $x = x_0$ when $t = 0$
x		$x = x_0 \sin \omega t$	$x = x_0 \cos \omega t$
v	$v = \pm\omega\sqrt{x_0^2 - x^2}$	$v = \omega x_0 \cos \omega t$	$v = -\omega x_0 \sin \omega t$
a	$a = -\omega^2 x$	$a = -\omega^2 x_0 \sin \omega t$	$a = -\omega^2 x_0 \cos \omega t$

Graphical Representation of Variation with Time



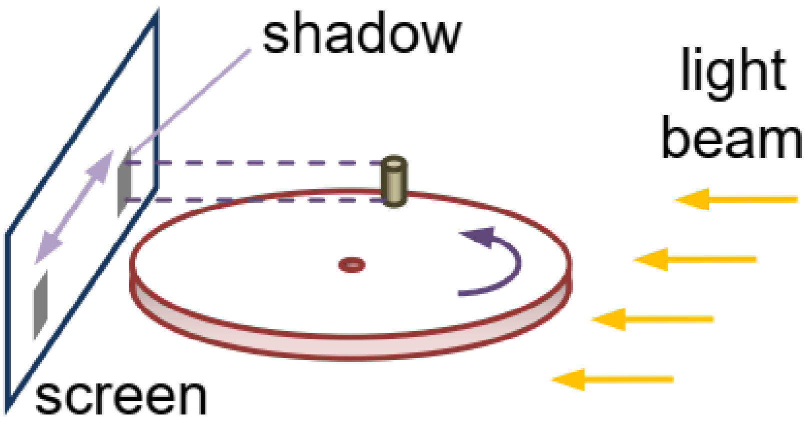


From the time variation of displacement, velocity and acceleration, it can be seen that they are out of phase with each other by either $\frac{\pi}{2}$ rad or π rad.

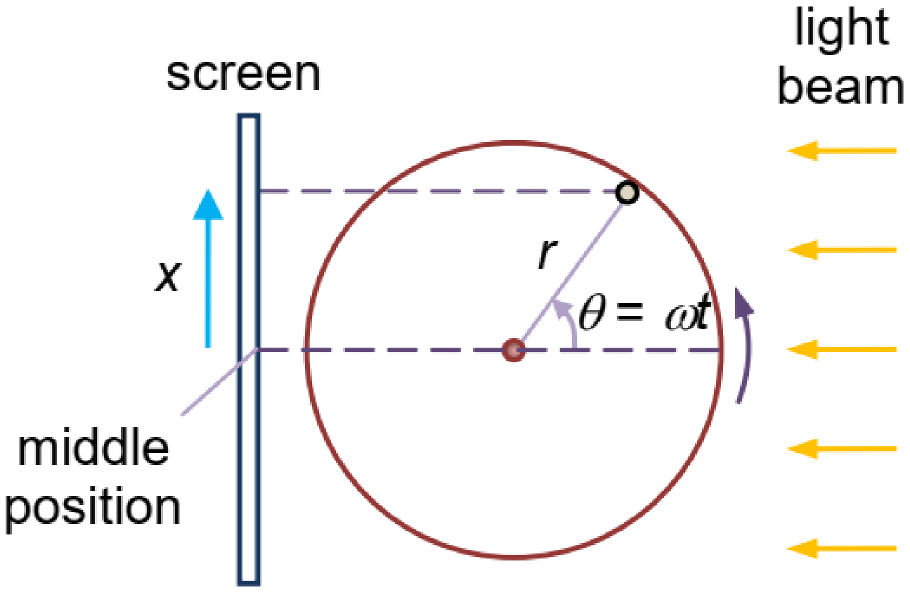


Uniform Circular Motion and S.H.M.

Shadow moves in SHM horizontally as peg on turntable undergoes uniform circular motion



Side view



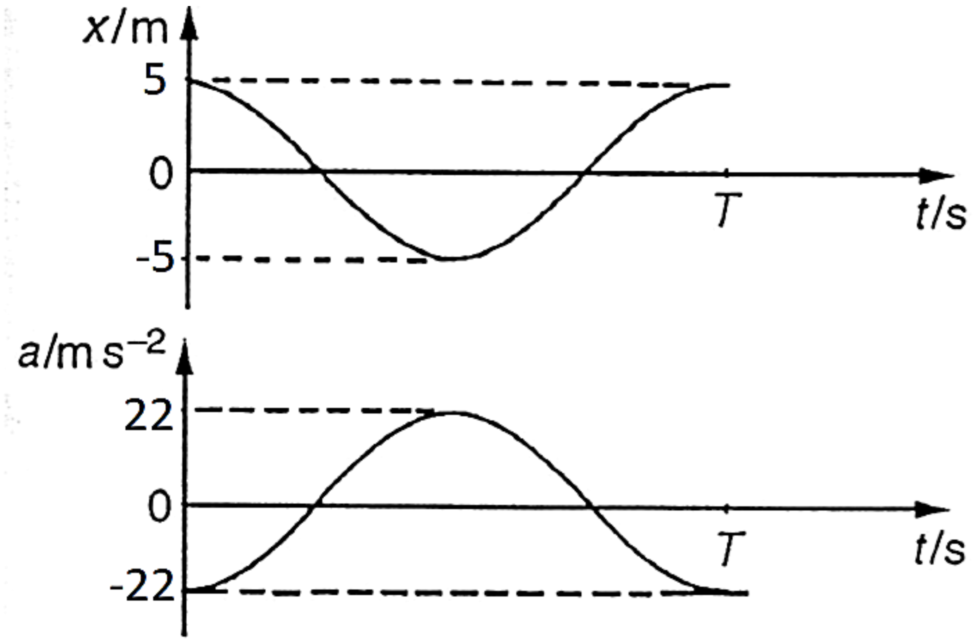
Plan view

The shadow's displacement from the middle position is $x = r\sin\omega t$ which is sinusoidal and thus the shadow's motion is SHM.

Practice Example 3

The time evolution of displacement x and acceleration a of an oscillator displaying simple harmonic motion are shown below.

What is the period of oscillation T?



Practice Example 4

A particle oscillates with simple harmonic motion along a line with a maximum speed v_0. What is the speed of the particle when its displacement a third of its amplitude?

Energy in Simple Harmonic Motion

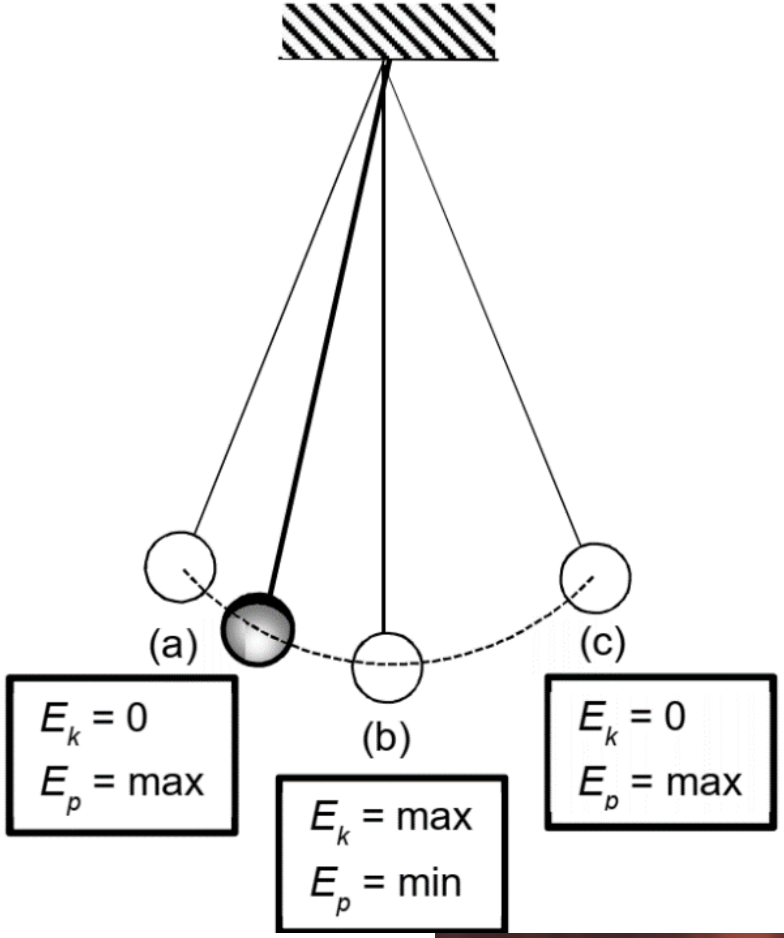
When a body is in S.H.M., there is a continuous interchange of its kinetic energy and potential energies. If the body oscillates with a constant amplitude, its total energy remains constant over time and displacement.

Let us consider the interchange of energy for the oscillations of a simple pendulum that has negligible resistance to its motion.

1. At the top of its swing (a), the pendulum possesses zero kinetic energy and maximum (gravitational) potential energy.

2. As the pendulum is releases from the top of its swing (a) and moves towards its equilibrium position (b), its (gravitational) potential energy is converted to kinetic energy.

3. At its equilibrium position (b), the pendulum possesses maximum kinetic energy and minimum (gravitational) potential energy

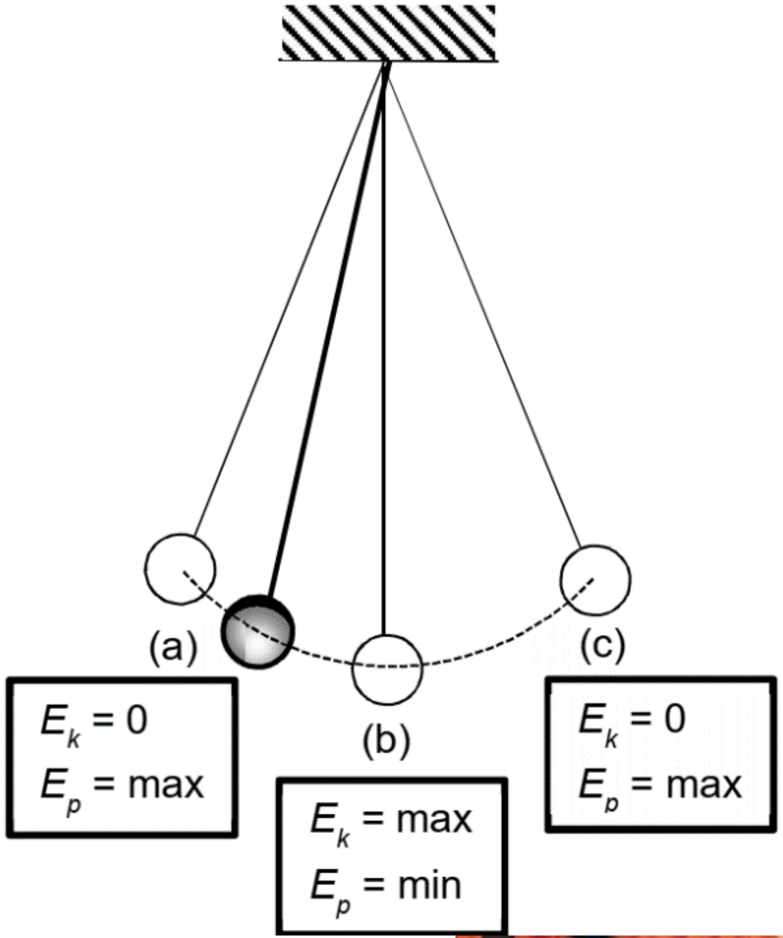


Energy in Simple Harmonic Motion

4. As the pendulum moves past its equilibrium position (b) and moves towards the other extreme position (c), its kinetic energy is converted to (gravitational) potential energy.

5. At the other extreme position (c), the pendulum possesses zero kinetic energy and maximum (gravitational) potential energy.

6. The energy conversion cycle repeats.



Note:
A similar energy conversion cycle can also be described for the oscillations of a vertical mass-spring system. However, it must be noted that potential energy will refer to the sum of both its gravitational and elastic potential energies

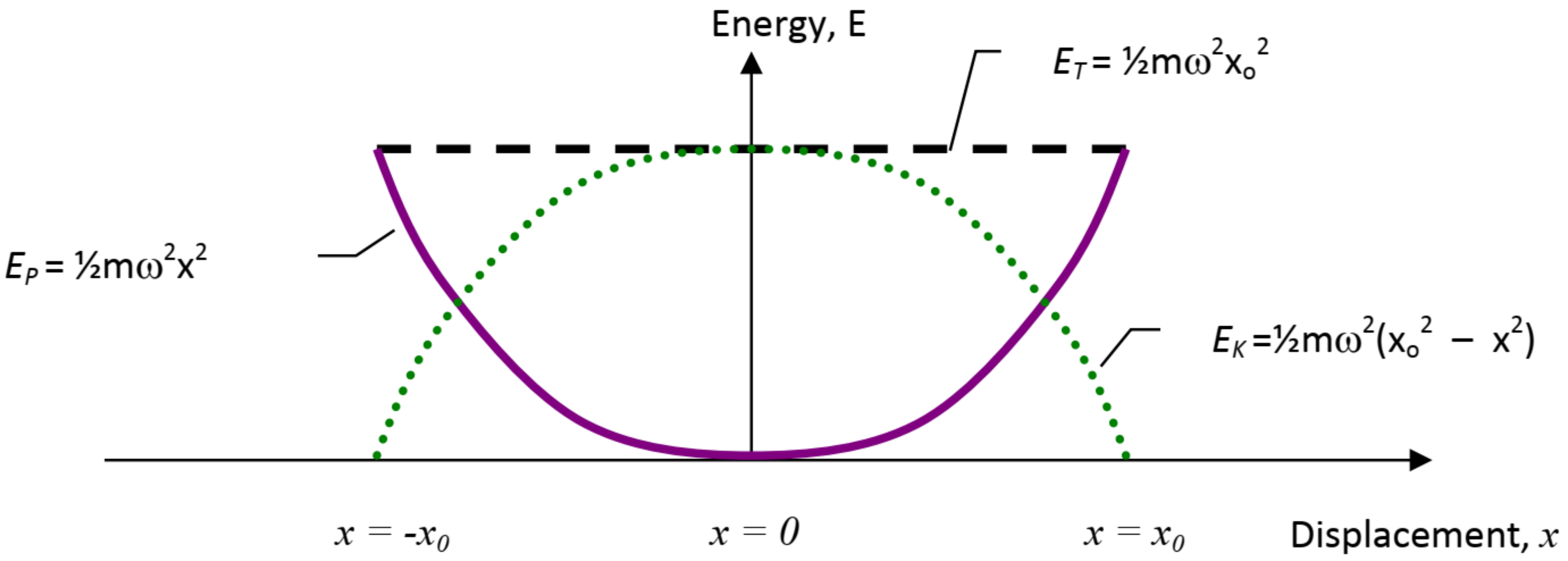
Variation of Energy with Displacement and Time

Energy is the capacity to do work. It exists around us in many different forms. Some of the common forms are shown below.

	Function of displacement (x)	Function of time
Kinetic Energy E_K	$\frac{1}{2}m\omega^2(x_0^2 - x^2)$	$\frac{1}{2}m\omega^2 x_0^2 \cos^2(\omega t)$
Potential Energy E_P	$\frac{1}{2}m\omega^2 x^2$	$\frac{1}{2}m\omega^2 x_0^2 \sin^2(\omega t)$
Total Energy E_T	$\frac{1}{2}m\omega^2 x_0^2$	

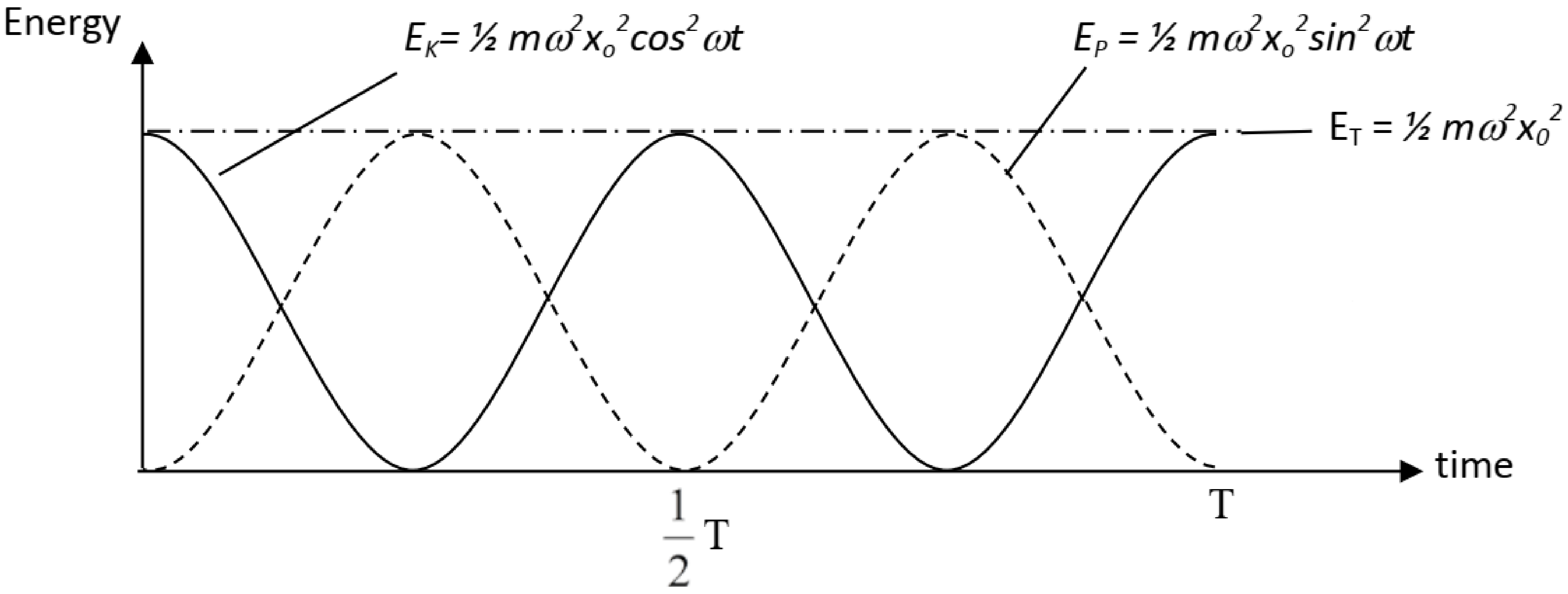
*derivations are found in the appendix

Graphical Representation of Various Energies with respect to Displacement



Note: When sketching these graphs involving displacement, take good care to ensure that the graphs do not extend beyond the extreme displacements (i.e. x_0 and $– x_0$).

Graphical Representation of Various Energies with respect to Time



Practice Example 5

An object in simple harmonic motion has a mass of 0.75 kg. Its displacement is governed by the equation

$$x = 2.50 \cos 3.6t$$

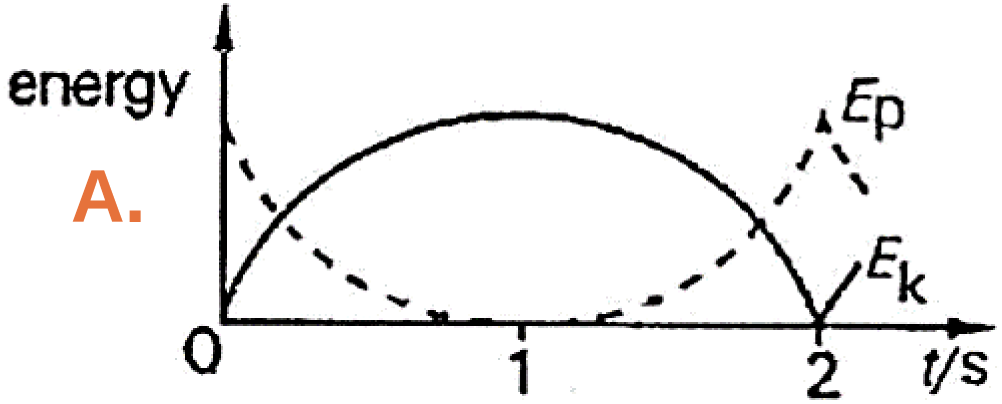
where x is in meters and t is in seconds.

a) What is the total energy of the object?
b) Determine the kinetic energy of the object at $t = 0.2\ s$?
c) What is the potential energy of the object at $t = 0.2\ s$?

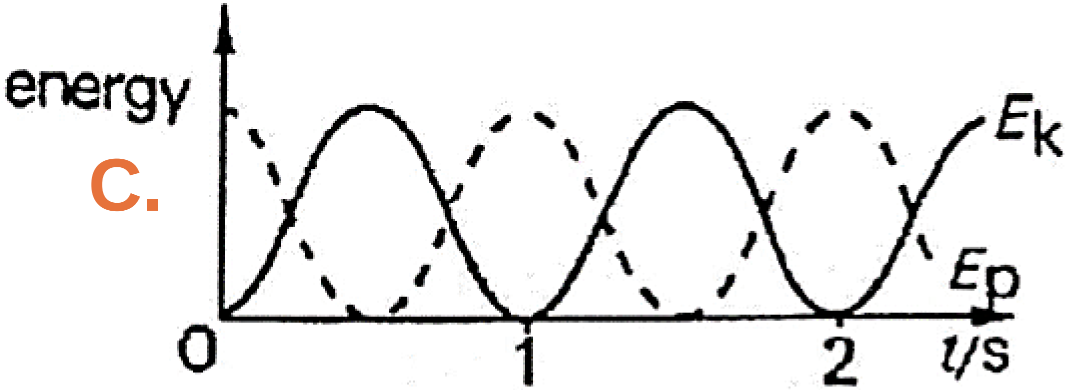
Practice Example 6

The bob of a simple pendulum of period 2s is given a small displacement and then released at $t = 0$s. Which diagram shows the variations with time of the bob's kinetic energy E_K and its potential energy E_P?

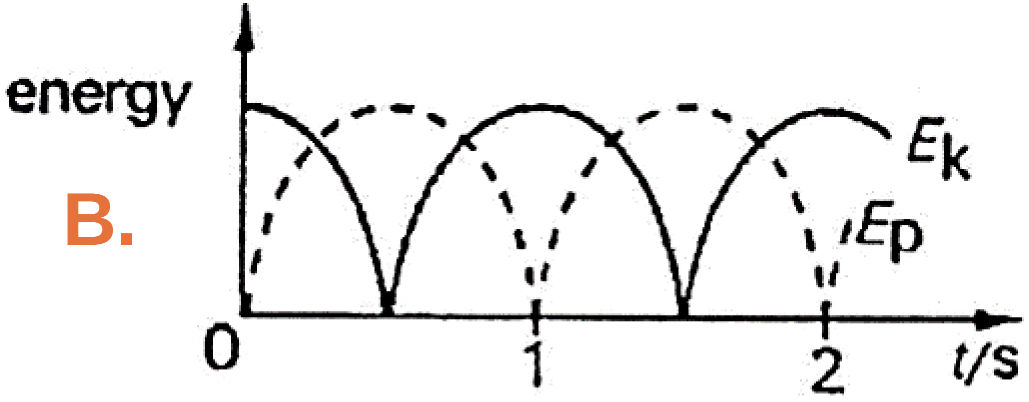
A.



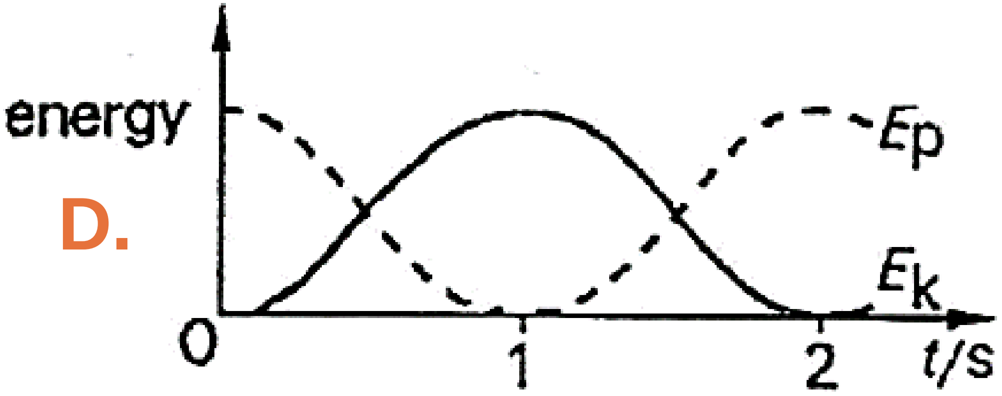
C.



B.



D.



DAMPED OSCILLATION

A damped oscillation is an oscillation that loses energy until it comes to a stop due to resistive forces

2 types of forces acting on the system:

a. Restoring force
b. Dissipative force

The rate if removal of energy from an oscillating system is called the degree of damping. There are 3 different degrees of damping:

1. Light Damping (Under-damped system)
2. Critical damping (Critically damped system)
3. Heavy Damping (Over-damped system

DAMPED OSCILLATION

<table>
<tr><th>Light Damping</th><th>Critical Damping</th><th>Heavy Damping</th></tr>
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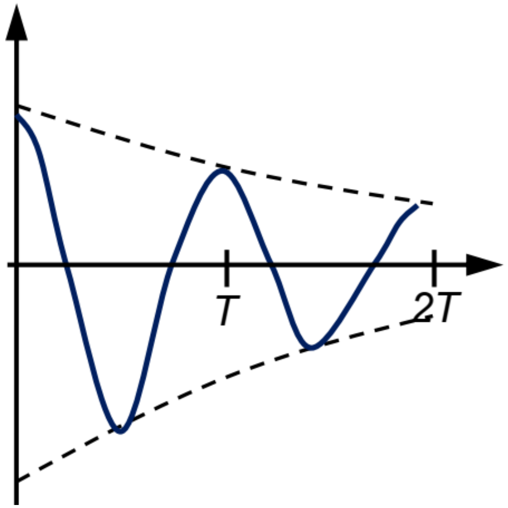
<td>
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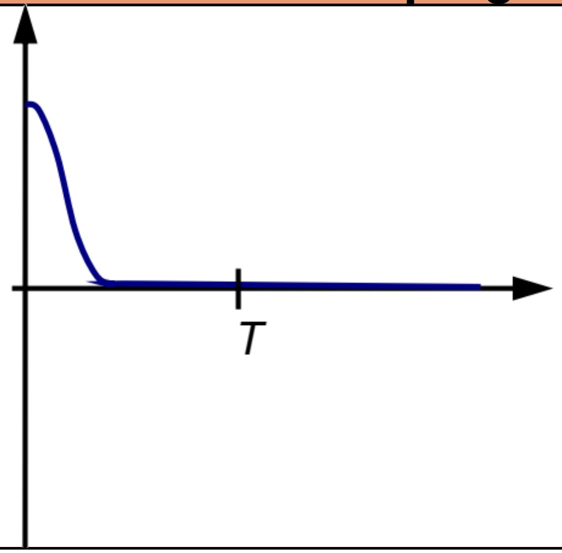
<td>
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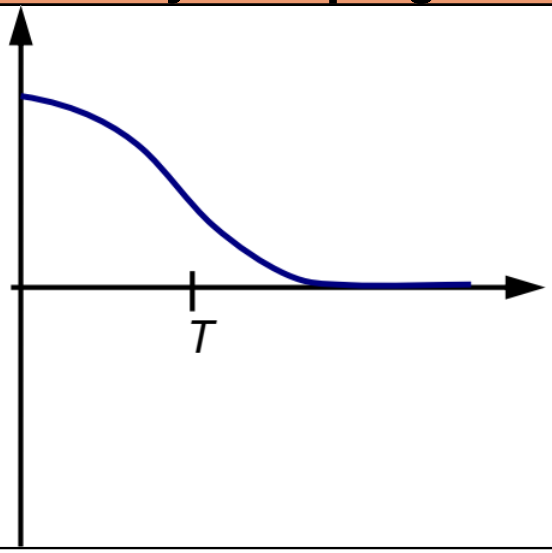
<td>
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<tr>
<td>▪ Amplitude will decrease gradually with time
▪ Period may remain almost the same
▪ e.g., an object oscillating in air/water</td>
<td>▪ body does not oscillate but returns to its equilibrium position in the shortest possible time
▪ e.g., car suspension system, needle of measuring instrument</td>
<td>▪ body will take a long time to return to its equilibrium position without oscillating.
▪ e.g., swing color</td>
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</table>

FORCED OSCILLATION

A forced oscillation is one which **an oscillator** is made to oscillate at the frequency of an external **driver.**

3 types of forces acting on the system:

a. Restoring force
b. Dissipative force
c. Driving force

Since the driver is causing the oscillator to oscillate, the driver is also transferring energy to the oscillator.

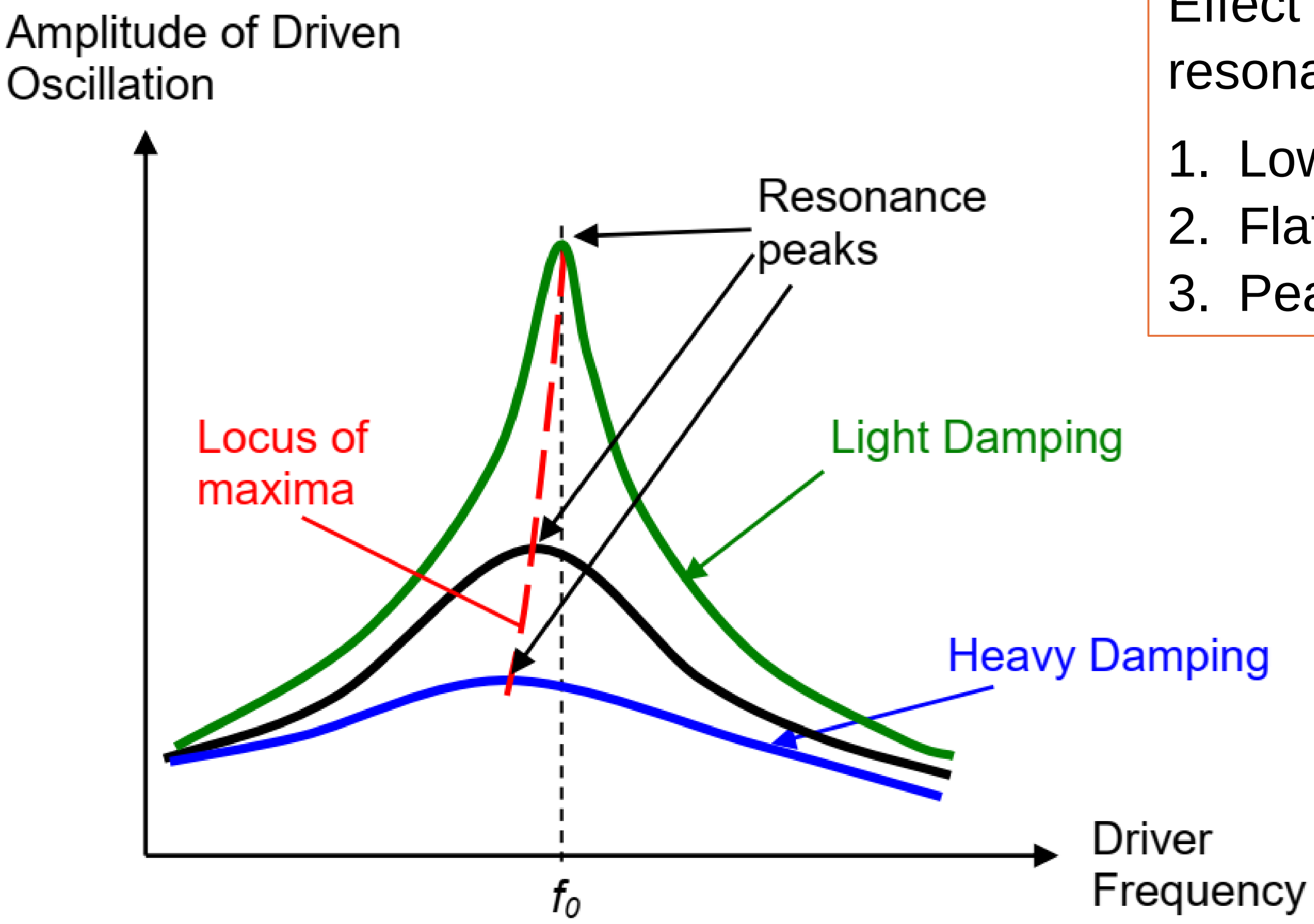
RESONANCE

- is the phenomenon in which there is a maximum transfer of energy from the driver to the oscillator.
- The oscillator achieves maximum amplitude
- This phenomenon occurs when the driving frequency matches the natural frequency of the oscillator

Effect of Damping on Resonance

- The degree of damping may be increased to reduce maximum amplitude at resonance

RESONANCE: Amplitude vs Frequency Curve



Effect of increased damping on resonance curve

1. Lower at all frequencies
2. Flatter peak
3. Peak skewed to the left

APPLICATIONS of RESONANCE

Useful Resonance	Destructive Resonance
Musical Instruments Resonance is responsible for the production of sound in many musical instruments especially the wind instruments.	**Shattering of Glass** It has been known for high-pitched sound waves to shatter fragile objects. For example, an opera singer hitting a top note may shatter a wine glass at resonance.
Radio Receptions A radio receiver works on the principle of resonance. Our air is filled with radio waves of many different frequencies which the aerial picks up. The tuner can be adjusted so that the frequency of the electrical oscillations in the circuits is the same as that of the radio waves transmitted from the particular station we desired. The effect of resonance amplifies the signals contained in this wave while the radio waves of other frequencies are diminished	**Earthquakes** During earthquakes, buildings are forced to oscillate in resonance with the seismic waves, resulting in serious damages. In regions of the world where earthquake happen regularly, buildings may be built on foundations that absorb the energy of the shock waves. In this way, the vibrations are damped and the amplitude of the oscillations cannot reach dangerous levels.

APPLICATIONS of RESONANCE

Useful Resonance	Destructive Resonance
Microwave Cooking In a microwave oven, microwaves with a frequency similar to the natural frequency of vibration of water molecules are used. When food containing water molecules is placed in the oven, the water molecules resonate, absorbing energy from the microwaves and consequently get heated up. This absorbed energy then spreads through the food and cooks it. The plastic or glass containers do not heat up since they do not contain water molecules.	**Human Internal Organs** In human beings, internal organs can be made to resonate in response to external frequencies, usually below 10 Hz. High levels of vibration can cause serious, or even fatal lung, heart, intestinal, and brain damage.
Magnetic Resonance Imaging Strong, varying radio frequency electromagnetic fields are used to cause oscillations in atomic nuclei. When resonance occurs, energy is absorbed by the molecules. By analyzing the pattern of energy absorption, a computer-generated image can be produced	

SUGGESTED SOLUTIONS TO PRACTICE EXAMPLES

Practice Example 1

A spring-mass system is vertically suspended from a ceiling. The acceleration of the block of mass m is given by

$$a = -\frac{k}{m}x$$

where x is the displacement of the block. Discuss why the block's motion is simple harmonic.

Answer:

The negative sign in front of x shows that the acceleration is opposite in direction to the displacement. Since $\frac{k}{m}$ is a constant, the acceleration of the motion is proportional to its displacement. Thus, the motion of the block is simple harmonic.

Practice Example 2

Consider two identical pendulums. One takes 26.0 seconds to complete 15 oscillations while the other one lags 1.23 seconds behind the first. Determine the phase difference between the pendulums.

Solution:

Period:

$$T = \frac{26}{15} = 1.73\ s$$

$$\Delta t = 1.73 - 1.23 = 0.5\ s$$

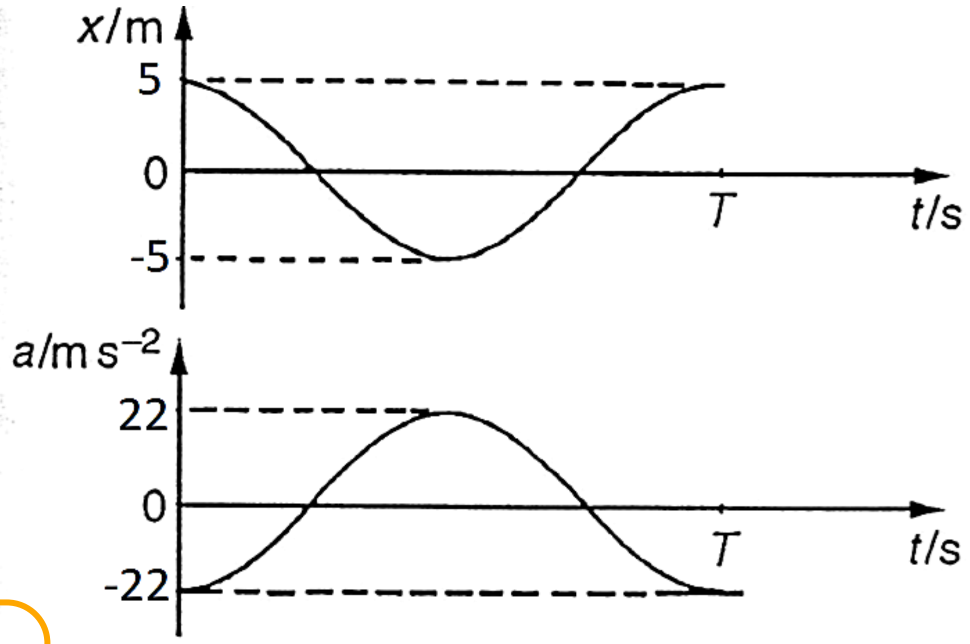
Phase Difference:

$$\Delta\phi = \frac{2\pi\Delta t}{T} = \frac{2\pi(0.5)}{1.73} = 1.82\ \text{rad}$$

Practice Example 3

The time evolution of displacement x and acceleration a of an oscillator displaying simple harmonic motion are shown below.

What is the period of oscillation T?



Solution:

At $t = 0$, $x = 5.0\ m$ and $a = -22\ m/s^2$

$$a = -\omega^2 x$$

$$a = -\left(\frac{2\pi}{T}\right)^2 x \rightarrow T = 2\pi\sqrt{\frac{-x}{a}}$$

$$T = 2\pi\sqrt{\frac{-5}{-22}} = 3.00\ s$$

Practice Example 4

A particle oscillates with simple harmonic motion along a line with a maximum speed v_0. What is the speed of the particle when its displacement a third of its amplitude?

Solution:

$$v = \omega\sqrt{x_0^2 - x^2}$$

$$v_0 = \omega x_0$$

$$v = \omega\sqrt{x_0^2 - \left(\frac{1}{3}x_0\right)^2} = \frac{2\sqrt{2}}{3}\,\omega x_0$$

$$v = \frac{2\sqrt{2}}{3}v_0 = 0.94v_0$$

Practice Example 5

An object in simple harmonic motion has a mass of 0.75 kg. Its displacement is governed by the equation

$$x = 2.50\cos 3.6t$$

where x is in meters and t is in seconds.

a) What is the total energy of the object?
b) Determine the kinetic energy of the object at $t = 0.2\ s$?
c) What is the potential energy of the object at $t = 0.2\ s$?

Solution (a):

$$x_0 = 2.50\ m$$
$$\omega = 3.6\ Hz$$
$$E_T = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2}(0.75)(3.6)^2(2.5)^2$$
$$E_T = 30.4\ J$$

Solution (c):

$$E_P = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}(0.75)(3.6)^2(1.88)^2$$
$$E_P = 17.2\ J$$

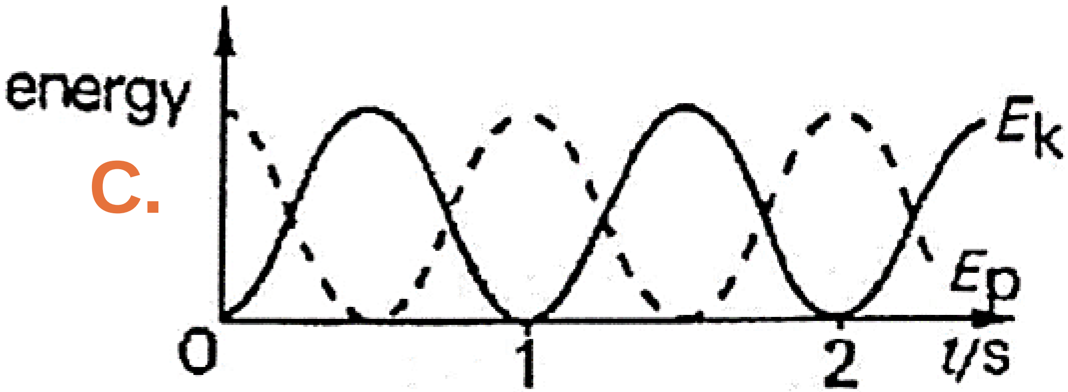
Solution (b):

$$x = 2.50\cos(3.6 \times 0.2)$$
$$x = 1.88\ m$$
$$E_K = \frac{1}{2}m\omega^2(x_0^2 - x^2) = \frac{1}{2}(0.75)(3.6)^2[2.5^2 - 1.88^2]$$
$$E_K = 13.2\ J$$

Practice Example 6

The bob of a simple pendulum of period 2s is given a small displacement and then released at $t = 0$s. Which diagram shows the variations with time of the bob's kinetic energy E_K and its potential energy E_P?

C.



Appendix

Variation of Energy with Displacement and Time

	Function of displacement (x)	Function of time
Kinetic Energy E_K	Substitute $v = \pm\omega\sqrt{x_0^2 - x^2}$ into $E_K = \frac{1}{2}mv^2$	If $x = x_0 \sin\omega t$ then $\frac{dx}{dt} = \omega x_0 \cos\omega t$. Substitute into $E_K = \frac{1}{2}mv^2$
Potential Energy E_P	$E_T = E_P + E_K \rightarrow E_P = E_T - E_K$ $E_P = \frac{1}{2}m\omega^2 x_0^2 - \frac{1}{2}m\omega^2(x_0^2 - x^2)$	Substitute $x = x_0 \sin\omega t$ into $E_P = \frac{1}{2}m\omega^2 x^2$
Total Energy E_T	$E_P = 0$ when $E_k =$ maximum (at $x = 0$) $E_T = E_{K,max} = \frac{1}{2}m\omega^2 x_0^2$ *holds true for all positions $\frac{1}{2}m\omega^2 x_0^2$	$E_T = E_P + E_K$ $E_T = \frac{1}{2}m\omega^2 \cos^2(\omega t) + \frac{1}{2}m\omega^2 x_0^2 \sin^2(\omega t)$