MARCH PRACTICE QUESTIONS 2021
SECONDARY 4 EXPRESS
SECONDARY 5 NORMAL ACADEMIC

ELEMENTARY MATHEMATICS **4048/01**

Specimen Paper **MARKING SCHEME**

Date: **3 March 2021** **Duration: NIL**

Candidates answer on separate writing paper

READ THESE INSTRUCTIONS FIRST

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For $\boldsymbol{\pi}$, use either your calculator value of $\mathbf{3.142}$, unless the question requires the answer in terms of $\boldsymbol{\pi}$.

Topic names will be listed above each question for your benefit and revision

Upon completion of solutions:

Each candidate have exactly 2 weeks to submit their solutions
Take a picture or send the digital version of your solutions to me (Kaiwen) via Telegram (**@kaiwen_tutor**) or WhatsApp (**90583779**)
Ensure that all workings are clear and legible
Solutions will be marked based on your presentation, accuracy and completeness of your solutions
A markers' report and the full solutions will be provided at the end of the month

Setter: Ong Kai Wen

This question paper consists of 9 printed pages including the cover page

Content Covered

- Algebraic Expressions and Formulae
- Set Language & Notation
- Congruency & Similarity
- Pythagoras' Theorem & Trigonometry
- Mensuration
- Properties of Circles
- Angles, Triangles & Polygons
- Algebraic Manipulation
- Coordinate Geometry
- Data Analysis & Handling
- Functions & Graphs
- Problems in Real-World Context (Algebra)

All questions are sourced and selected based on the known abilities of students sitting for the 'O' Level E-Math Examination. If questions are sourced from respective sources, credit will be given when appropriate

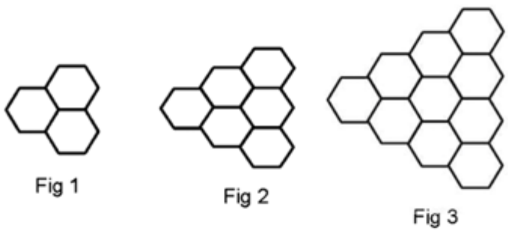
Special Note from Tutor (Kaiwen):

Some of these questions are slightly more challenging than others and require some out of the box thinking. When faced with such challenging questions, always go back to the fundamentals and think about the basics you already have learnt in school. Questions will never deviate away from the curriculum that is already pre-set for you

Nonetheless, don't give up if you are unable to solve the questions! Send in your solutions as how you would submit your answer scripts during the National Examinations. From there, I will be able to see and judge the ability of the cohort before moving on and planning the curriculum and content for the rest of the year.

All the best and I really do hope that this initiative will help as many students as it can reach! 加油!

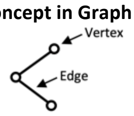
Topic: Algebraic Expressions & Formulae



Mary has used satay sticks to form groups of hexagons like the one shown before. She tabulates her findings in a table and tried to find a pattern with the formatted groups

Figure	Number of sticks (S)	Number of hexagons formed (H)	Number of vertices* formed after joining sticks (V)
1	15	3	13
2	27	6	22
3	42	10	33
4	a	b	c

Remarks / Hint
*** Vertices (plural for Vertex) represents the endpoints of an edge [From the concept in Graph Theory]**



(a) Write down the values of a, b and c
(b) Form, and write down the equation connecting S, H and V
(c) Using the formula, find the value of S when $H = 66$ and $V = 166$
(d) State a reason, why the number 400 cannot appear in the S column
(e) Find the 8^{th} term of the sequence in the S column
(f) Find the formula of the n^{th} term of the sequence in the H column

Solution

(a)

$a = (42 - 27) + (4 - 1) + 42 = 60$

Explanation **To quickly find the number of sticks used, we use the earlier given figures. We take Figure 3 - Figure 2** $(42 - 27)$ **to get the remaining last row of sticks used. We then take the last row, and additional 4 sticks to make up the last hexagon of the figure to form the last row of Figure 4.** $(42 - 27 + 4)$ **Finally, we add the last row of Figure 4 and the original Figure 3 to form Figure 4.** $(42 - 27) + 4 + 42$ **However, we remove 1 stick as there will be an overlap when adding the figures together.** $(42 - 27) + (4 - 1) + 42$

Overlapping stick

$\boldsymbol{b = (10 - 6) + 1 + 10 = 15}$

<u>*Explanation*</u> To quickly find the number of Hexagons present, we use the earlier given figures. We take Figure 3 - Figure 2 to get the last row of Figure 3. $(10 - 6)$ We add the last row of Figure 3, add 1 extra hexagon to form Row 5 of Figure 4 and add back the original Figure 3 to form Figure 4. $(10 - 6) + 1 + 10$

Overlapping Vertex

$\boldsymbol{c = (33 - 22) + (3 - 1) + 33 = 46}$

<u>*Explanation*</u> To quickly find the number of Vertices present, we use the earlier figures given. Similar to above, we take Figure 3 - Figure 2 $(33 - 22)$ to get the number of vertices in the last row. We then take the last row, and add additional 3 vertices to make up the last hexagon of the figure to form the last row of Figure 4. $(33 - 22) + 3$ Finally, we add the last row of Figure 4 and the original Figure 3 to form Figure 4. $(33 - 22) + 3 + 33$ However, we remove 1 vertex as there will be an overlap when adding the figures together $(33 - 22) + (3 - 1) + 33$

(b) By observing the table, $S = H + V - 1$

(c) $S = 66 + 166 - 1 = 231$

(d) S must be a multiple of 3. However, 400 is not a multiple of 3

(e) By observing patterns, we can see that the difference between terms increases by 3, starting with a difference of 12

$S_1 = 15$
$S_2 = 15 + 12 = 27$
$S_3 = 27 + 12 + 3 = 42$
$S_4 = 42 + 12 + 2(3) = 60$
$S_5 = 60 + 12 + 3(3) = 81$
$S_6 = 81 + 12 + 4(3) = 105$
$S_7 = 105 + 12 + 5(3) = 132$
$S_8 = 132 + 12 + 6(3) = 162$

8^{th} term has 162 sticks used

(f) For sequence $3, 6, 10, \ldots$

We look at the differences between successive terms:

Let H_n be the term number, d_n be the difference between each term:

$$H_2 - H_1 = d_1 \Rightarrow 6 - 3 = 3$$
$$H_3 - H_2 = d_2 \Rightarrow 10 - 6 = 4$$
$$H_4 - H_3 = d_3 \Rightarrow 15 - 10 = 5$$
$$H_5 - H_4 = d_4 \Rightarrow 21 - 15 = 6$$

Hence, the next few terms will be:

$$H_5 + d_5 = H_6 \Rightarrow 21 + 7 = 28$$
$$H_6 + d_6 = H_7 \Rightarrow 28 + 8 = 36$$
$$H_7 + d_7 = H_8 \Rightarrow 36 + 9 = 45$$

Hence, from the pattern above, the sequence can be written as:

$$H_1 = 3$$
$$H_2 = 3 + [3]$$
$$H_3 = 3 + [3 + 4]$$
$$\ldots$$
$$H_n = 3 + [3 + 4 + \cdots + (n + 1)]$$

Let's evaluate the terms in the square bracket! Let the sum of the square bracket be T_n

There are total $(n - 1)$ terms in the square bracket

$$T_{n-1} = 3 + 4 + 5 + \cdots + (n + 1) \ldots\ldots\ldots\ldots\ldots (1)$$

We can write equation (1) backwards:

$$T_{n-1} = (n + 1) + \cdots + 5 + 4 + 3 \ldots\ldots\ldots\ldots\ldots (2)$$

Add both Equation (1) and (2) together to give:

$$2\,T_{n-1} = (n + 4) + \cdots + (n + 4) + (n + 4) + (n + 4) = (n - 1)(n + 4)$$

Hence:

$$T_{n-1} = \frac{(n - 1)(n + 4)}{2}$$

$$\therefore H_n = 3 + \frac{(n - 1)(n + 4)}{2}$$
$$= \frac{6 + n^2 + 3n - 4}{2}$$
$$= \frac{n^2 + 3n + 2}{2}$$
$$= \frac{1}{2}(n + 1)(n + 2)$$

Topic: Set Language & Notation

An activity camp offers **3** sports: Tennis, Cricket and Volleyball. One day, **50** children took part in these sports. **19** children played tennis, **34** played cricket, **23** played volleyball. **2** children played all three sports. **10** children played tennis and volleyball. **5** children played tennis and cricket

(a) Draw a Venn Diagram to represent all the information above. Label all regions of the Venn Diagram and insert all necessary information to make the solution complete

(b) Find the number of children who played

(i) Tennis and cricket but not volleyball

(ii) Cricket and volleyball but not tennis

(iii) Cricket only

There was a typo in the question! It should be **10** children playing tennis and volleyball and not **5**. For the solutions I have marked, I will just follow whatever the question with the error intended. For other students just reviewing the solutions, please take note of the change!

(a) (i) $n(\textbf{Tennis only}) = 19 - 3 - 2 - 8 = 6$

$n(\textbf{Tennis and Cricket only}) = 5 - 2 = 3$

$n(\textbf{Tennis and Volleyball only}) = 10 - 2 = 8$

$n(\textbf{Cricket and Volleyball only}) = a$

$n(\textbf{Cricket only}) = 5 + a + b = 34 \Rightarrow a + b = 29 \dots\dots\dots (1)$

$n(\textbf{Volleyball only}) = 10 + a + c = 23 \Rightarrow a = 13 - c \dots\dots\dots (2)$

$\textbf{Total number of students} = 19 + a + b + c = 50 \Rightarrow a + b + c = 31 \dots\dots\dots (3)$

Substitute Equation (1) into Equation (3),

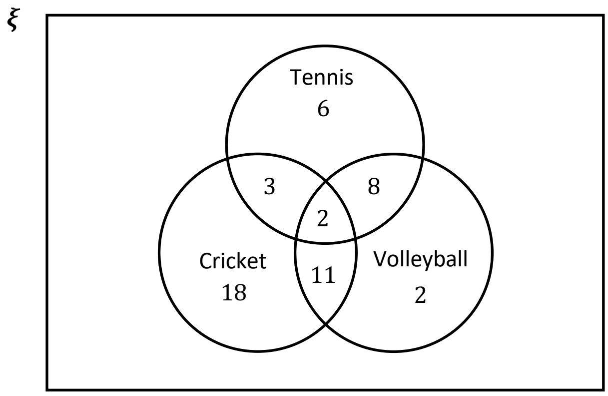
$(29) + c = 31 \Rightarrow c = 2$

Substitute $c = 2$ into Equation (2),

$a = 13 - (2) = 11$

Substitute $a = 11$ into Equation (1),

$(11) + b = 29 \Rightarrow b = 18$

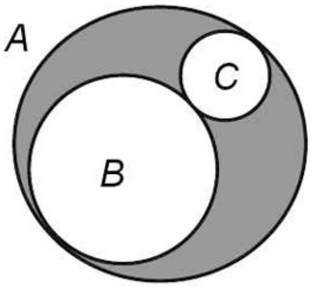


(ii) a) $n(\textbf{Tennis and Cricket but not Volleyball}) = 3$

b) $n(\textbf{Cricket and Volleyball but not Tennis}) = 11$

c) $n(\textbf{Cricket}) = 18$

Topic: Congruency and Similarity



A designer wishes to create a photo frame for his clients as shown in the diagram. In the diagram, $\mathbf{2}$ smaller circles, $\boldsymbol{B}$ and $\boldsymbol{C}$ are enclosed within a bigger circle $\boldsymbol{A}$. The radius of the bigger circle $\boldsymbol{A}$ is $\mathbf{8\ cm}$. He wishes to know the rough measurements of his clients' blueprints. Given that the radius of circle $\boldsymbol{B}$ is $\mathbf{6\ cm}$, calculate the ratio of the sum of areas of the $\mathbf{2}$ smaller circles to the area of the shaded region

Solution

$$\begin{aligned}\textbf{Radius of circle } \boldsymbol{C} &= \frac{\mathbf{2(8) - 2(6)}}{\mathbf{2}} \\ &= \frac{\mathbf{16 - 12}}{\mathbf{2}} \\ &= \mathbf{2\ cm}\end{aligned}$$

$$\begin{aligned}\frac{\textbf{Area of 2 smaller circles}}{\textbf{Area of circle } \boldsymbol{A}} &= \frac{\boldsymbol{\pi}\mathbf{(6)^2} + \boldsymbol{\pi}\mathbf{(2)^2}}{\boldsymbol{\pi}\mathbf{(8)^2}} \\ &= \frac{\mathbf{36 + 4}}{\mathbf{64}} \\ &= \frac{\mathbf{40}}{\mathbf{64}} \\ &= \frac{\mathbf{5}}{\mathbf{8}}\end{aligned}$$

$$\begin{aligned}\frac{\textbf{Area of 2 smaller circles}}{\textbf{Area of shaded region}} &= \frac{\mathbf{5}}{\mathbf{8 - 5}} \\ &= \frac{\mathbf{5}}{\mathbf{3}}\end{aligned}$$

Topic: Algebraic Expressions & Formulae

A man drove from Town A to Town B and then returned to Town A. The distance between Town A and B is approximately $\mathbf{720\ km}$

(a) During the journey from Town A to Town B, the man drove at an average speed of x $\mathbf{km/h}$. Write down an expression, in terms of x, for the time taken for the journey

(b) On the return journey, the man found that his average speed was reduced by $\mathbf{10\ km/h}$ due to a delay caused by an accident. Write down an expression, in terms of x, for the time taken for the journey

(c) Given that the difference in time for the $\mathbf{2}$ journeys was $\mathbf{45\ min}$, form an equation in x and show that it reduces to

$$x^2 - 10x - 9600 = 0$$

(d) Solve the equation $x^2 - 10x - 9600 = 0$, giving your answers correct to $\mathbf{1}$ decimal place

(e) Find the time required for the return journey

Solution

(a) Time taken $= \dfrac{720}{x}$ hrs

(b) Time taken for the return journey $= \dfrac{720}{x-10}$ hrs

(c) Since the time difference between the 2 journeys is 45 min

$$\frac{720}{x-10} - \frac{720}{x} = \frac{45}{60}$$

$$\frac{720x - 720(x-10)}{x(x-10)} = \frac{3}{4}$$

$$\frac{7200}{x^2 - 10x} = \frac{3}{4}$$

$$3x^2 - 30x = 28800$$

$$x^2 - 10x - 9600 = 0 \text{ (shown)}$$

(d) $x^2 - 10x - 9600 = 0$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-9600)}}{2(1)}$$

$$x = \frac{10 + \sqrt{38500}}{2} \quad \text{or} \quad x = \frac{10 - \sqrt{38500}}{2}$$

$$x = 103.107\ldots \qquad x = -93.107\ldots$$

$$x = 103.1 \text{ (1.d.p.)} \qquad x = -93.1 \text{ (1.d.p.)}$$

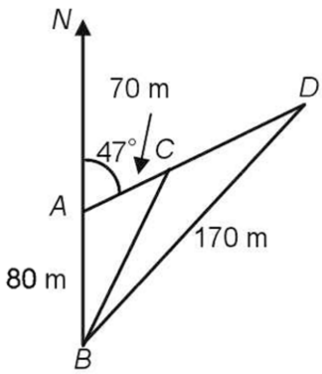
(e) $x = 103.1 \qquad x = -93.1$ (N.A.)

$$\text{Time taken for the return journey} = \frac{720}{\left(\frac{10 + \sqrt{38500}}{2}\right) - 10}$$

$$= 7.73303\ldots$$

$$= 7.73 \text{ hrs}$$

Topic: Trigonometry



The diagram shows $\mathbf{4}$ points, $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ on the ground with $\boldsymbol{D}$ at a bearing of $\mathbf{047°}$ from $\boldsymbol{A}$. Point $\boldsymbol{B}$ is $\mathbf{80\ m}$ due south of $\boldsymbol{A}$. Point $\boldsymbol{C}$ is on the line $\boldsymbol{AD}$ such that it is $\mathbf{70\ m}$ from $\boldsymbol{A}$. $\boldsymbol{BD}$ is $\mathbf{170\ m}$

(a) Find

(i) The distance $\boldsymbol{BC}$

(ii) $\angle \boldsymbol{ADB}$

(iii) The bearing of $\boldsymbol{D}$ from $\boldsymbol{B}$

(b) There is a tree at point $\boldsymbol{A}$. A boy measures the angle of elevation of the top of the tree from point $\boldsymbol{B}$ and finds that it is $\mathbf{23°}$. Find the height of the tree

(c) The boy walks from point $\boldsymbol{B}$ to point $\boldsymbol{D}$ along the line $\boldsymbol{BD}$ until he reaches a point $\boldsymbol{X}$ which is nearest to $\boldsymbol{A}$. Find the distance $\boldsymbol{BX}$

Solution

(a) **(i)** $\angle BAC = 180° - 47°$

$= 133°$ (**supplementary angles**)

By cosine rule,

$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos\angle BAC$

$BC^2 = 80^2 + 70^2 - 2(80)(70)\cos 133°$

$BC = \sqrt{11300 - 11200\cos 133°}$

$= 137.616\ldots$

$= 138\ \text{m}\ (3.\text{s.f.})$

(ii) **By sine rule,**

$$\frac{AB}{\sin\angle ADB} = \frac{BC}{\sin\angle BAD}$$

$$\frac{80}{\sin\angle ADB} = \frac{170}{\sin 133°}$$

$$\sin\angle ADB = \frac{80\sin 133°}{170}$$

$$\angle ADB = \sin^{-1}\left(\frac{80\sin 133°}{170}\right)$$

$= 20.130\ldots$

$= 20.1°\ (1.\text{d.p.})$

(iii) Bearing of D from $B = 180° - \angle ADB - \angle BAC$

$$= 180° - \sin^{-1}\left(\frac{80 \sin 133°}{170}\right) - 133°$$

$$= 47 - \sin^{-1}\left(\frac{80 \sin 133°}{170}\right)$$

$$= 026.869 \ldots$$

$$= 026.9° \ (1. \text{d. p.})$$

(b) Let E be the top of the tree

$$\tan \angle EBA = \frac{AE}{AB}$$

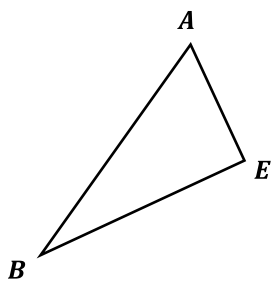
$$AE = 80 \tan 23°$$

$$= 33.957 \ldots$$

$$= 34.0 \text{ m } (3. \text{s. f.})$$

(c) To find the nearest point to A, it is the shortest perpendicular distance from A to BD

Shortest distance from A to BD can be found by drawing a right-angled triangle



Let the point that coincides to the shortest distance on BD be E

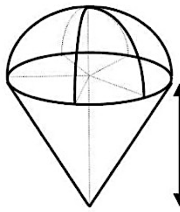
$$\cos \angle ABE = \frac{BE}{AB}$$

$$BE = 80 \cos\left(47° - \sin^{-1}\left(\frac{80 \sin 133°}{170}\right)\right)$$

$$= 71.363 \ldots$$

$$= 71.4 \text{ m } (3. \text{s. f.})$$

Topic: Mensuration



The figure above is made up of a solid metallic cone of diameter $\mathbf{27\ cm}$ and height $\mathbf{35\ cm}$ and a solid hemisphere. Taking $\boldsymbol{\pi} = \frac{22}{7}$, calculate

(a) The mass of the solid, if the density of the metal used is $\mathbf{0.7\ g/cm^3}$ (give your answer in $\mathbf{kg}$)

(b) The total surface area of the solid, correct to the nearest $\mathbf{cm^2}$

(c) The new radius, if the original figure is melted down and reshaped into a sphere

Solution

(a) **Radius of cone** $= \mathbf{13.5\ cm}$

$$\text{Volume of cone portion} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\left(\frac{22}{7}\right)(13.5)^2(35)$$

$$= 6682\frac{1}{2}\ \text{cm}^3$$

$$\text{Volume of hemispherical portion} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\left(\frac{22}{7}\right)(13.5)^3$$

$$= 5155\frac{1}{14}\ \text{cm}^3$$

$$\text{Total volume of solid} = 6682\frac{1}{2} + 5155\frac{1}{14}$$

$$= 11837\frac{4}{7}\ \text{cm}^3$$

$$\therefore \text{Mass of solid} = \frac{\left(11837\frac{4}{7}\right)(0.7)}{1000}$$

$$= 8.2863\ldots$$

$$= 8.29\ \text{kg}\ (3.\text{s.f.})$$

(b) **Let l be the slant height of the cone**

$$\boldsymbol{l = \sqrt{35^2 + 13.5^2}}$$

$$\boldsymbol{= \sqrt{1407\frac{1}{4}}\ \text{cm}}$$

$$\boldsymbol{\therefore \text{Curved surface area of cone} = \pi r l}$$

$$\boldsymbol{= \left(\frac{22}{7}\right)(13.5)\left(\sqrt{1407\frac{1}{4}}\right)}$$

$$\boldsymbol{= 42\frac{3}{7}\left(\sqrt{1407\frac{1}{4}}\right)\ \text{cm}^2}$$

$$\boldsymbol{\text{Curved surface area of hemisphere} = 2\pi r^2}$$

$$\boldsymbol{= 2\left(\frac{22}{7}\right)(13.5)^2}$$

$$\boldsymbol{= 1145\frac{4}{7}\ \text{cm}^2}$$

$$\boldsymbol{\therefore \text{Total surface area of solid} = 42\frac{3}{7}\left(\sqrt{1407\frac{1}{4}}\right) + 1145\frac{4}{7}}$$

$$\boldsymbol{= 2737.208\ldots}$$

$$\boldsymbol{= 2737\ \text{cm}^2\ (\text{nearest cm}^2)}$$

(c) **We equate the volume of the solid to the volume of a sphere to find the radius**

$$\boldsymbol{11837\frac{4}{7} = \frac{4}{3}\pi r^3}$$

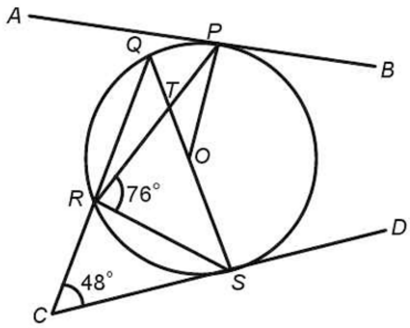
$$\boldsymbol{11837\frac{4}{7} = \frac{4}{3}\left(\frac{22}{7}\right)r^3}$$

$$\boldsymbol{\therefore r = \sqrt[3]{\frac{\left(11837\frac{4}{7}\right)}{\left(\frac{88}{21}\right)}}}$$

$$\boldsymbol{= 14.1362\ldots}$$

$$\boldsymbol{= 14.1\ \text{cm}\ (3.\text{s.f.})}$$

Topic: Properties of Circles



$\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{R}$ and $\boldsymbol{S}$ are $\mathbf{4}$ points on the circle with the centre $\boldsymbol{O}$. $\boldsymbol{QS}$ is a diameter of the circle. $\boldsymbol{AB}$ is the tangent to the circle at $\boldsymbol{P}$ and $\boldsymbol{CO}$ is the tangent to the circle at $\boldsymbol{S}$. $\boldsymbol{PR}$ and $\boldsymbol{QS}$ intersect the circle at $\boldsymbol{T}$. Given that $\boldsymbol{CRQ}$ is a straight line, $\angle \boldsymbol{PRS} = \mathbf{76}°$ and $\angle \boldsymbol{RCS} = \mathbf{48}°$, calculate

(a) $\angle \boldsymbol{SQC}$

(b) $\angle \boldsymbol{ROQ}$

(c) $\angle \boldsymbol{RPB}$

Solution

(a) $\angle OSC = 90° \ (CD \text{ is tangent to the circle at } S)$

$\angle SQC = 180° - 90° - 48°$

$= 42° \ (\text{angles in a triangle})$

(b) $OQ = OR \ (\text{radius of the circle})$

$\therefore \Delta ORQ$ **is an isosceles triangle**

$\angle ROQ = 180° - 42° - 42°$

$= 96° \ (\text{angles in an isosceles triangle})$

(c) $\angle QRS = 90° \ (\text{angles in a semicircle})$

$\angle QRT = 90° - 76°$

$= 14° \ (\text{complementary angle})$

$\angle POQ = 14° \times 2$

$= 28° \ (\text{angle at centre} = 2 \times \text{angle at circumference})$

$\angle QTR = 180° - \angle RQT - \angle QRT$

$= 180° - 42° - 14°$

$= 124° \ (\text{angles in a triangle})$

$\angle QTR = \angle PTO \ (\text{vertically opposite angles})$

$\angle TPO = 180° - \angle PTO - \angle POT$

$= 180° - 28° - 124°$

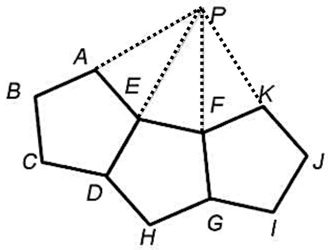
$= 28° \ (\text{angles in a triangle})$

$\angle OPB = 90° \ (AB \text{ is tangent to the circle at } P)$

$\therefore \angle RPB = 90° + 28°$

$= 118°$

Topic: Angles, Triangles & Polygons



The diagram shows 3 regular pentagons $ABCDE$, $DEFGH$ and $FGIJK$. BA, DE and GF are produced to meet P

(a) Find

- **(i)** $\angle AED$
- **(ii)** $\angle AEB$
- (iii) $\angle AEF$

(b) Show that triangles APE and FPE are congruent

(c) Explain, with working, why BEF is a straight line

(d) Additional pentagons are added to the 3 in the diagram above to form a closed ring surrounding a regular polygon.

- **(i)** Find the total number of pentagons needed to form the ring
- **(ii)** What is the name for this ring formed

Solution

(a) **(i)** $\angle AED$ **corresponds to 1 interior angle of the pentagon**

$$\therefore \angle AED = \frac{(n-2)\times 180°}{n}$$
$$= \frac{(5-2)\times 180°}{5}$$
$$= 108°$$

(ii) **Since** $ABCDE$ **is a regular polygon,** ΔAEB **is an isosceles triangle**

$$\therefore \angle AEB = \frac{180° - 108°}{2}$$
$$= 36°$$

(iii) $\angle AEF = 360° - \text{reflex } \angle AEF$

$$= 360° - 2(108°)$$
$$= 144°$$

(b) PE **is a common side**

$AE = EF$ (**sides of 2 identical regular polygons**)

$AP = FP$ (**both are lines extending towards a common point** P)

Hence, by the SSS congruency test, ΔAPE **and** ΔFPE **are congruent**

(c) $\angle AEB = 36°$

$\angle AEF = 144°$

Hence, these 2 angles sum up to $180°$ **which implies that** BEF **is colinear and a straight line**

(d) (i) Exterior angle of this new polygon $= \angle AEB$

$$= 36°$$

$$\therefore \text{Number of sides of this polygon} = \frac{360°}{36°}$$

$$= 10$$

Therefore, the total number of polygons needed to form this ring is 10

(ii) Decagon

Topic: Algebraic Manipulation

If $\frac{1}{m} - \frac{1}{n} = 5$, find the value of

$$\frac{3m + 5mn - 3n}{m - 3mn - n}$$

Solution

$\frac{1}{m} - \frac{1}{n} = 5$

$n - m = 5mn$

$m - n = -5mn \dots\dots\dots (1)$

$\frac{3m + 5mn - 3n}{m - 3mn - n} = \frac{3(m - n) + 5mn}{(m - n) - 3mn} \dots\dots\dots (2)$

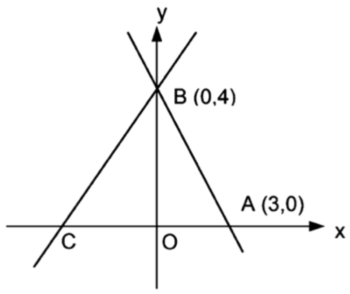
Substitute Equation (1) into (2),

$$\frac{3(m - n) + 5mn}{(m - n) - 3mn} = \frac{3(-5mn) + 5mn}{(-5mn) - 3mn}$$

$$= \frac{-10mn}{-8mn}$$

$$= \frac{5}{4}$$

Topic: Coordinate Geometry



The diagram shows $\mathbf{2}$ straight lines $\boldsymbol{AB}$ and $\boldsymbol{BC}$. The equation of the straight line $\boldsymbol{BC}$ is $\mathbf{5y - 4x = 20}$, $\boldsymbol{A}$ and $\boldsymbol{B}$ are the points $(\mathbf{3, 0})$ and $(\mathbf{0, 4})$ respectively. Find

(a) The coordinates of $\boldsymbol{C}$

(b) The equation of $\boldsymbol{AB}$

(c) The area of $\boldsymbol{\Delta ABC}$

Solution

(a) **At $\boldsymbol{C}$, $\boldsymbol{y = 0}$**

$\therefore \mathbf{5(0) - 4x = 20}$

$\mathbf{-4x = 20}$

$\mathbf{x = -5}$

$\therefore \boldsymbol{C}(\mathbf{-5, 0})$

(b) **To find the gradient,**

$$\textbf{Gradient of } \boldsymbol{AB} = \frac{\mathbf{4-0}}{\mathbf{0-3}}$$

$$= -\frac{\mathbf{4}}{\mathbf{3}}$$

$\boldsymbol{y}$**-intercept** $= \mathbf{4}$

$$\therefore \boldsymbol{y} = -\frac{\mathbf{4}}{\mathbf{3}} + \mathbf{4}$$

(c) **To find the area,**

$$\textbf{Area of triangle } \boldsymbol{ABC} = \frac{\mathbf{1}}{\mathbf{2}} \times \mathbf{8} \times \mathbf{4}$$

$$= \mathbf{16\ units^2}$$

Topic: Data Analysis & Handling

The following are marks obtained by 24 students in a Mathematical test

32	20	44	21	43	34	36	22
55	30	59	40	26	37	35	51
33	51	49	51	58	45	36	46

(a) Draw a stem-leaf diagram to illustrate the results

(b) Find the median mark

(c) The pass mark is 50. The teacher decided to moderate the marks by multiplying each of the students' mark by 2 and then subtracting 20 from the result

(i) Calculate the increase in the number of students who passed after the moderation of marks

(ii) John's mark did not change after the moderation. What was his mark?

Solution

(a)

Stem	Leaf							
2	0	1	2	6				
3	0	2	3	4	5	6	6	7
4	0	3	4	5	6	9		
5	1	1	1	5	8	9		

Legend

$2 \mid 0 = 20$

(b) **The median is calculated by using the 12th and 13th mark**

$$\therefore \text{Median mark} = \frac{37+40}{2}$$

$$= 38.5$$

(c) **(i)** **Let x be the unknown mark**

$2x - 20 = 50$

$x = 35$

Hence, the initial mark must be 35 and above to be considered a pass after the moderation

Number of new people that pass $= 10$

(ii) **Let y be John's mark**

Since there is no change in the mark,

$2y - 20 = y$

$y = 20$

Topic: Functions & Graphs

Psychologist have discovered that, theoretically, the students ability $(\boldsymbol{y})$ to absorb concepts in the classroom and the time taken to explain the concepts $(\boldsymbol{t}\ \textbf{mins})$ are related by the expression

$$\boldsymbol{y = -0.1t^2 + 2.6t + 43}$$

(a) What is the student's ability to absorb concepts in the classroom after $\mathbf{7}$ minutes?
(b) Find the time when the student's ability is $\mathbf{48}$
(c) By completing the square, rewrite the above expression in the form $\boldsymbol{-0.1(t-a)^2+b}$
(d) The time taken to explain a concept is optimum when the students' ability to absorb is the highest. The expression is sketched on a graph. What is the optimum time to explain a concept? Which point of the graph does this time correspond to?
(e) Find the time when the students are so saturated that they cannot take in any more concepts

Solution

(a) **When $t = 7$,**

$$y = -0.1(7)^2 + 2.6(7) + 43$$
$$= 56.3$$

(b) **When $y = 48$,**

$$48 = -0.1t^2 + 2.6t + 43$$
$$0.1t^2 - 2.6t + 5 = 0$$
$$t^2 - 26t + 50 = 0$$
$$t = \frac{-(-26) \pm \sqrt{(-26)^2 - 4(1)(50)}}{2(1)}$$
$$= \frac{26 \pm \sqrt{476}}{2}$$
$$t = 23.90871\ldots \quad \text{or} \quad t = 2.09128\ldots$$
$$t = 23.9\ (3.\text{s.f.}) \quad \text{or} \quad t = 2.09\ (3.\text{s.f.})$$

(c)

$$-0.1t^2 + 2.6t + 43 = -0.1\left(t^2 - 26t - 430\right)$$
$$= -0.1\left(t^2 - 2(13)(t) + 13^2 - 599\right)$$
$$= -0.1(t-13)^2 + 59.9$$

(d) **The optimum time to explain a concept is when $t = 13$**
This corresponds to the highest point/maximum point of the graph

(e) **Students are saturated when $y = 0$**

$$-0.1(t-13)^2 + 59.9 = 0$$
$$(t-13)^2 = 599$$
$$t - 13 = \pm\sqrt{599}$$
$$t = \pm\sqrt{599} + 13$$
$$t = 37.47447\ldots \quad \text{or} \quad t = -11.47447\ldots$$
$$t = 37.5\ (3.\text{s.f.}) \quad \text{or} \quad t = -11.5\ (3.\text{s.f.})\ (\text{NA})$$

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Topic: Problems in Real-World Context (Algebra)

A man wants to buy a car that costs $\$72\,000$. He has two types of car loan scheme to choose

(a) In loan scheme $\boldsymbol{A}$, he has to pay 15% of the total cost as the down payment and the rest as instalments for a period of 7 years at a fixed interest rate of 3.5% per year. Find

  (i) The amount he has to pay as down payment

  (ii) The total amount he has to pay for the car under scheme $\boldsymbol{A}$

(b) In loan scheme $\boldsymbol{B}$, he has to pay 25% of the total cost of the car as down payment and the balance in equal monthly instalments of $\$950$ over a period of 7 years

  (i) Calculate the total amount he has to pay under scheme $\boldsymbol{B}$

  (ii) Calculate the difference between scheme $\boldsymbol{A}$ and $\boldsymbol{B}$

(c) The man chooses the cheaper loan scheme

  (i) Find the additional cost incurred when he takes up the loan instead of paying $\$72\,000$

  (ii) Express the additional cost as a percentage of the selling price of the car, leaving your answer to 1 decimal place

Solution

(a) (i) To find out the amount he has to pay

$$\textbf{Down payment} = \$72\,000 \times 15\%$$
$$= \$10\,800$$

(ii) To find the total amount he has to pay in Scheme $\boldsymbol{A}$, we need to find the interest

$$\textbf{Amount of interest} = [(\$72\,000 - \$10\,800) \times 3.5\%] \times 7$$
$$= \$14\,994$$
$$\therefore \textbf{Total amount he has to pay} = \$72\,000 + \$14\,994$$
$$= \$86\,994$$

(b) (i) Total amount he has to pay in Scheme $\boldsymbol{B}$ is equivalent to the down payment plus the interest

$$\textbf{Down payment} = \$72\,000 \times 25\%$$
$$= \$18\,000$$
$$\textbf{Amount of interest} = \$950 \times 7 \times 12$$
$$= \$79\,800$$
$$\therefore \textbf{Total amount he has to pay} = \$18\,000 + \$79\,800$$
$$= \$97\,800$$

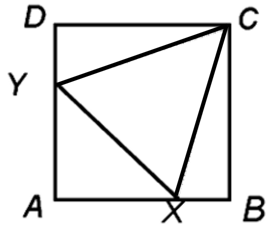
(ii) Difference in Scheme $\boldsymbol{A}$ and $\boldsymbol{B}$ $= \$97\,800 - \$86\,994$
$$= \$10\,806$$

(c) (i) Additional cost incurred $= \$86\,994 - \$72\,000$
$$= \$14\,994$$

(ii) To find the percentage,

$$\textbf{Additional cost incurred as a percentage of the selling price} = \frac{14\,994}{72\,000} \times 100\%$$
$$= 20.825$$
$$= 20.8\%\ (1.\text{d.p.})$$

Topic: Pythagoras' Theorem & Trigonometry



$\boldsymbol{ABCD}$ is a square of side $\mathbf{12}$ metres. Point $\boldsymbol{X}$ and $\boldsymbol{Y}$ are taken on $\boldsymbol{AX = AY = x}$ **m**

(a) Prove that the area of the triangle $\boldsymbol{CXY}$ is given by the expression below

$$\left(12x - \frac{1}{2}x^2\right) \text{ m}^2$$

(b) If the area of the triangle $\boldsymbol{CXY}$ is $\mathbf{70\ m^2}$, find the value of $\boldsymbol{x}$

Solution

(a) **To find the area of ΔCXY,**

$$\Delta CXY = \text{Area of } ABCD - \Delta AXY - \Delta CDY - \Delta CBX$$

$$\begin{aligned} \text{Area of } ABCD &= 12^2 \\ &= 144 \text{ m}^2 \end{aligned}$$

$$\text{Area of } \Delta AXY = \frac{1}{2}x^2 \text{ m}^2$$

$$\begin{aligned} \text{Area of } \Delta CDY &= \text{Area of } \Delta CBX \\ &= \frac{1}{2}(12)(12 - x) \\ &= (72 - 6x) \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \therefore \Delta CXY &= 144 - \frac{1}{2}x^2 - 2(72 - 6x) \\ &= 144 - \frac{1}{2}x^2 - 144 + 12x \\ &= \left(12x - \frac{1}{2}x^2\right) \text{ m}^2 \text{ (shown)} \end{aligned}$$

(b) $\text{Area of } \Delta CXY = 70 \text{ m}^2$

$$\left(12x - \frac{1}{2}x^2\right) = 70$$

$$x^2 - 24x + 140 = 0$$

$$(x - 10)(x - 14) = 0$$

$$x = 10 \quad \text{or} \quad x = 14 \text{ (rej } \because \text{ square is of length 12 m)}$$

End of Paper ☺