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TOPIC 2: KINEMATICS

THE ABOUT

CHAPTER ANALYSIS

MASTERY

- Be comfortable with 4 Kinematic equations
- Master Speed-time, Velocity-time, Acceleration-Time graphs
- Heavy calculation chapter

EXAM

- Projectile motion commonly tested
- Relate directly to Free-body diagrams to determine RF and thus Acceleration

WEIGHTAGE

- High overall weightage
- Tested consistently every year
- Typically a 7-10m question

KEY CONCEPT

BASIC KINEMATIC CONCEPTS
LINEAR MOTION WITH CONSTANT ACCELERATION
PROJECTILE MOTION

BASIC KINEMATIC CONCEPTS

TERMINOLOGY

SPEED (v or u)

SPEED is the rate of change of **distance.**

Velocity (v or u)

Velocity is the rate of change of displacement

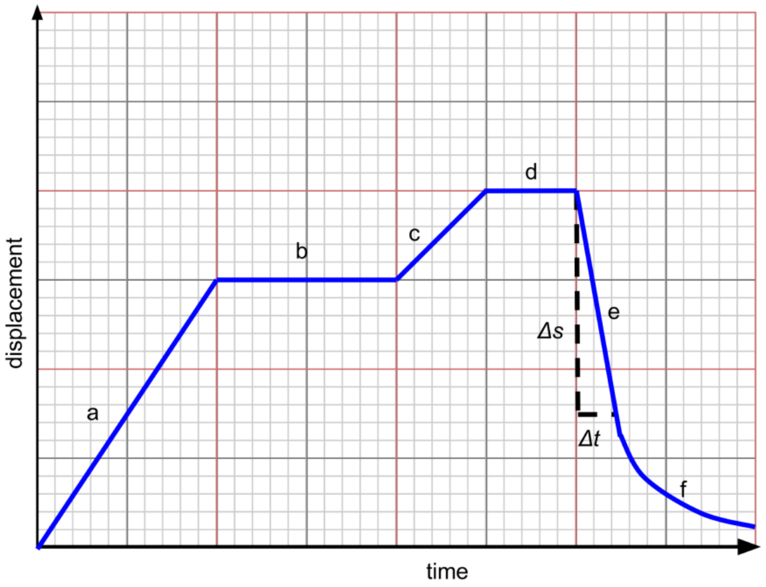
Acceleration (a)

Acceleration is the rate of change of velocity

Displacement (s)

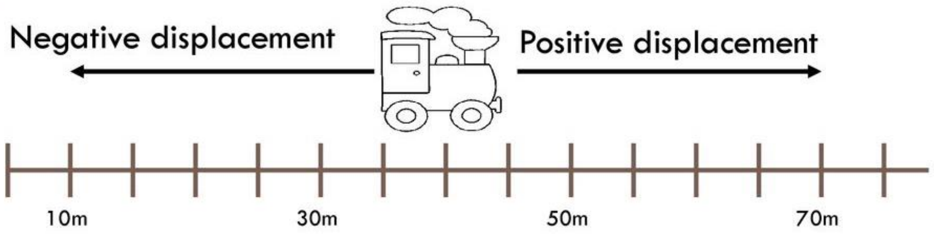
Displacement is the distance between the initial and final points of an object, **measured in a straight line in a specific direction.**

DISPLACEMENT TIME GRAPHS



Key features:

- Gradient of a displacement time graph represents **velocity**.
- The **area** under a velocity time graph represents **displacement** of an object
- Displacement is a **vector**, positive and negative values indicate **opposite directions**



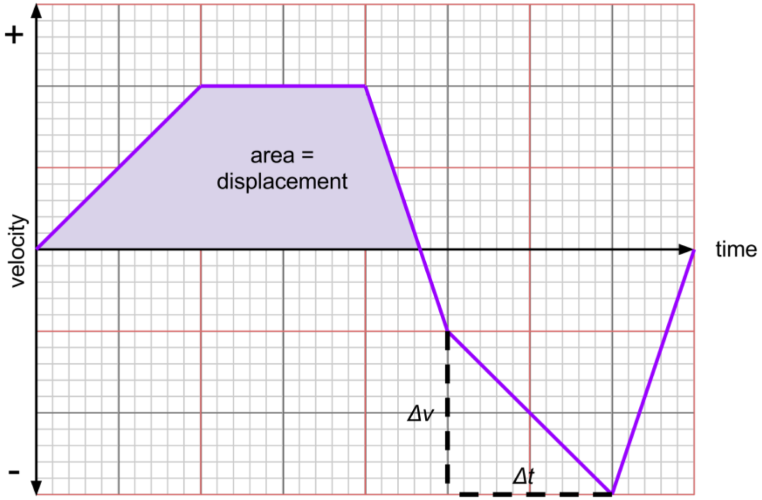
$x_f - x_i = 10m - 40m = -30m$ $x_f - x_i = 70m - 40m = 30m$

VELOCITY TIME GRAPHS

Key features:

- Gradient of a velocity - time graph represents **acceleration**.
- Velocity is a **vector**, positive and negative values indicate **opposite directions**

$$v = \frac{\Delta s}{\Delta t}$$



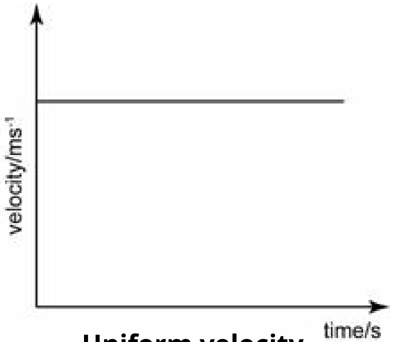
EXAM SKILL:

- Wherever kinematics is tested, always **sketch out a velocity time graph** where relevant, along with a diagram to aid in visualisation

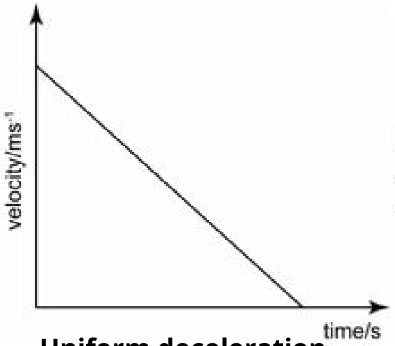
IMPORTANT VT GRAPHS

*Varying VT graphs are always tested with reference to **acceleration**

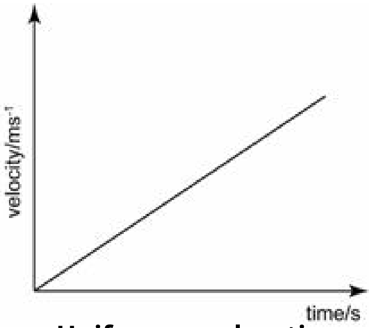
You must know how to sketch all of the following graphs



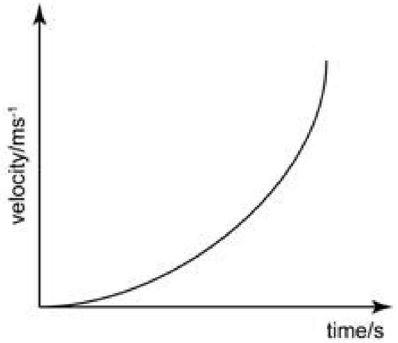
Uniform velocity



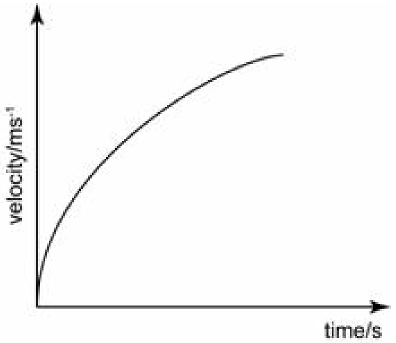
Uniform deceleration



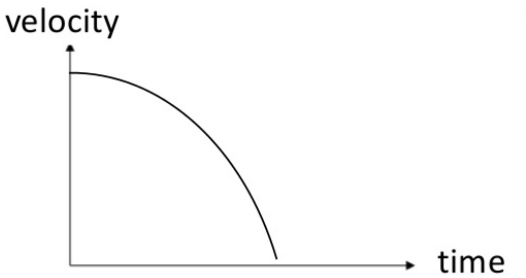
Uniform acceleration



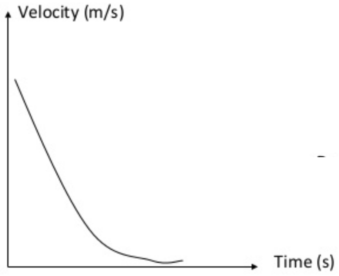
Increasing acceleration



Decreasing acceleration



Increasing deceleration



decreasing deceleration

UNIFORM ACCELERATION

When acceleration is **uniform**, the **4 kinematic equations** can be used

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

Typical Assumptions for constant acceleration:

1. *An object experiences no air resistance*
2. *The object is only under the influence of gravity; i.e: It is experiencing free fall.*

DETERMINING ACCELERATION

Acceleration can be determined by considering the **resultant force** acting on an object.

For an object with constant mass, by N2L:

$$Rf = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\left(\frac{dv}{dt}\right) = ma$$

Deriving further,

$$a = \frac{Rf}{m}$$

*In order to derive the magnitude of this acceleration, a **FREE BODY DIAGRAM,** is essential to calculating the magnitude of this resultant force.

*Direction of acceleration = Direction of change in momentum = Direction of resultant force

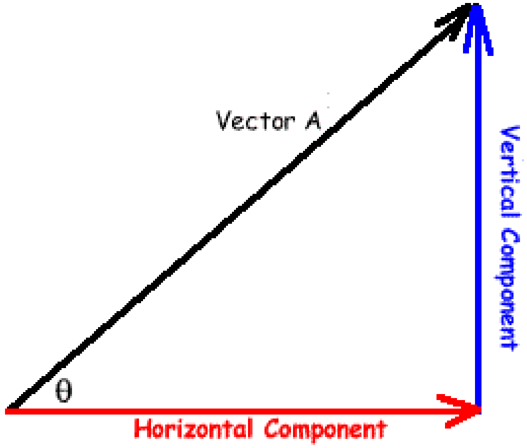
ACCELERATION DUE TO FREE FALL

When a mass falls **only due to the influence of gravity,** then Weight is the resultant force.

$$Rf = W$$
$$ma = mg$$
$$a = g$$
$$= 9.81\ m/s^2$$

Thus, an object will accelerate at the gravitational field strength constant of $9.81 m/s^2$

RESOLVING VECTOR INTO COMPONENTS



*Any 2 dimensional vector (I.e: Velocity, Force, Acceleration) can be resolved into its constituent **components.***

- *When moving into the angle, multiply by **cos of the angle***
- *When out of the angle, multiply by **sin of the angle***

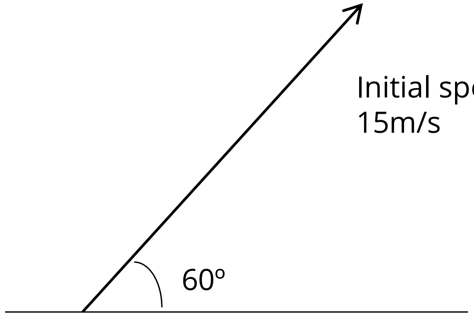
<u>Example</u>

A ball is thrown from the ground at an Angle of 60 degrees from the horizontal.

Determine its

i) initial horizontal speed

ii) Initial vertical speed



i) For initial horizontal speed, the **vector moves <u>into the angle</u>**

thus, $U_x = 15 \cos 60^\circ$

$= 7.5$ m/s

ii) For initial vertical speed, the vector moves out of the angle

Thus, $Uy = 15 \sin 60^\circ$

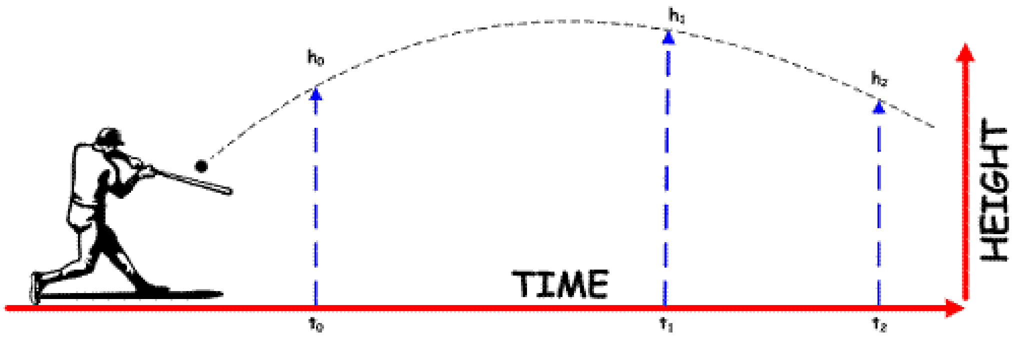
$= 12.99$

$= 13$ m/s

KEY CONCEPT

PROJECTILE MOTION

PROJECTILE MOTION



Projectile motion

- The study of motion of an object **in 2 axes (i.e: 2 directions).**
- These 2 directions are often **vertical and horizontal**
- If Air resistance is **negligible,** the path of the object is **parabolic**

KEY ACTIONS TO TAKE

- Split motion into 2 axes
- Clearly define a particular direction as **positive** and the opposite as **negative**
- Determine unknown value & known values (Refer to Tricks & tips)
- Apply 4 kinematic formulae to solve for unknowns

Projectile motion Tricks & Tips

Often tested concepts & implications:

1. **Maximum height**
   - At the maximum height of the projectile motion, **vertical component of velocity is 0.**
   - Thus, v = 0.

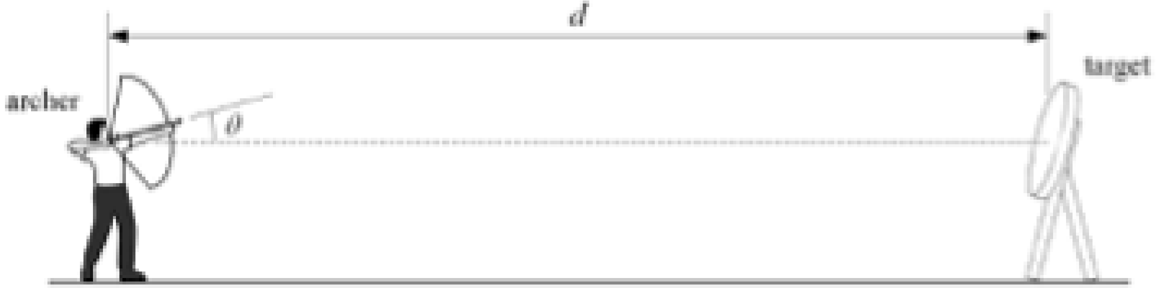
2. **Free fall**
   - When an object falls freely, Air resistance is negligible and **acceleration is constantly downwards at 9.81 m/s²**.

3. **Total time of flight**
   - For an object experiencing constant downward acceleration, **the time taken to fly upwards is equal to the time taken to fall to its starting point.**

4. **Maximum horizontal displacement**
   - Maximum range horizontally can be found by **first calculating** the **time of flight.** Given no horizontal resistive forces (Air Resistance =0), maximum horizontal displacement = $U_x t$

PROJECTILE MOTION

Example:



An Archer fires an arrow with initial speed *v* at an angle θ to the horizontal, hitting a target of distance d metres away, at the same height.

i) Given that the time of flight is t, and acceleration of freefall is *g*, write an expression for the following:

1. *d in terms of θ , v and t.*
2. *t in terms of v, θ and g*

ii) State the assumptions made in your calculations

Solution:

i)Consider motion only in the horizontal direction,

1. d = horizontal velocity x time of flight
   = u_x t
   = (v cos θ^o *t) metres*

2. Consider motion only in the vertical direction,
   at time t: since height is the same, Sy = 0

   Under only the influence of gravity,
   a = g

   U_y = V sin θ

   Find **t**

Considering 4 kinematics equation, **only the formula contains all listed quantities**

$$s = ut + \frac{1}{2}at^2$$

Subbing:

$$0 = \left(V \sin\theta \, t + \frac{1}{2} g \, t^2\right)$$

$$t = 0 \;\; (rej) \quad or \quad V sin\theta + ½ \, gt = 0$$

$$t = \frac{-2vsin\theta}{g}$$

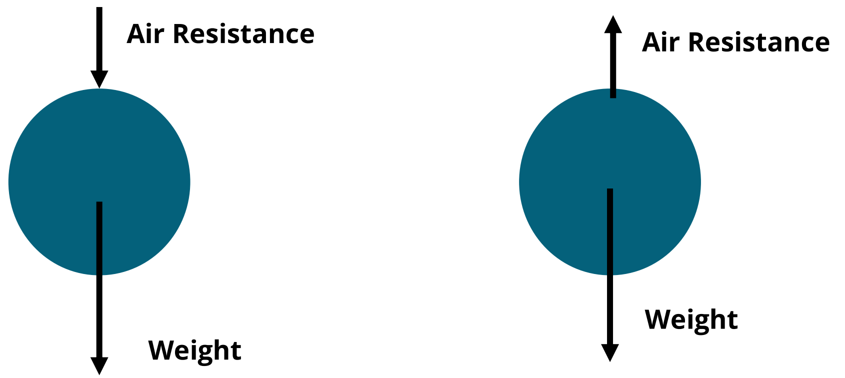
ii) Assumption, **no other forces such as air resistance** acts on the ball, causing the ball to <u>**fall freely due to gravity**</u>

EFFECTS OF AIR RESISTANCE ON TRAVEL TIME

In reality, **air resistance** always opposes the motion of objects.

*Consider a ball thrown upwards in earth's atmosphere where air resistance is **non negligible.***

*Why does the ball take a **shorter time to** reach maximum height than to fall from its maximum height to its original position?*



Ball Flying upwards

RF = W + AR (downward)

Ball Falling

RF = W - AR (downward)

Solution:

As the ball flies upwards, **Air resistance and weight both act downward**. Thus, **RF = W + AR.**

$$ma = mg + AR$$

$$\text{Thus, } a>g \implies a > 9.81 m/s^2 \text{ downwards}$$

However, as the ball falls down, Air resistance acts **upwards** while weight acts **downwards.**

Thus, **Rf= W-AR**

$$ma = mg - AR$$

$$\text{Thus, } a < g \implies a < 9.81 \ m/s^2 \text{ downwards}$$

Since the **distance travelled upwards to max height and distance travelled down to original position is the same,** the average speed of the ball as it travels **upwards** is always larger than its average speed travelling **downwards.**

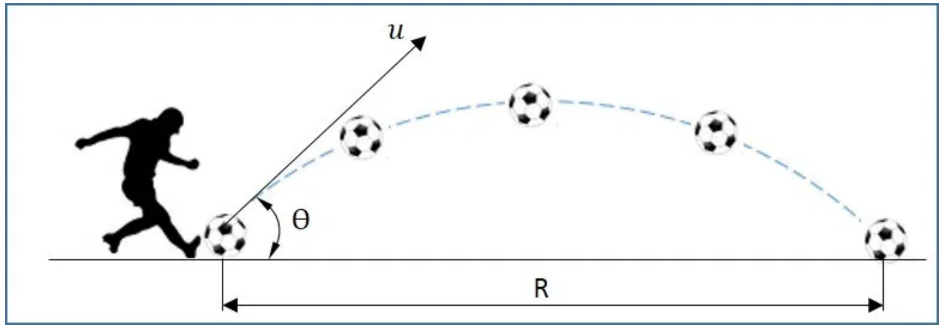
Hence, the time taken to travel up to max height is less than the time taken to travel downwards.

EFFECTS OF AIR RESISTANCE ON PATH

In reality, **air resistance** always opposes the motion of objects.

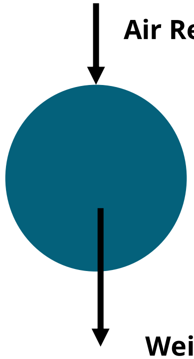
*Consider a ball undergoing a theoretical **parabolic projectile motion** without air resistance. How would **air resistance** alter its path?*

*Sketch the path of such a ball **with and without** the effects of Air Resistance*



<u>**Solution:**</u>

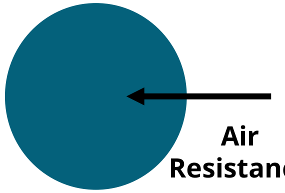
Consider vertical path & its resultant force **with air resistance**



RF = AR + W
Thus, ma = mg + AR
a>g, and **the object decelerates more quickly to rest,** <u>**Reaching a lower max height.**</u>

Consider horizontal path & its resultant force **with air resistance.**

The ball will experience **deceleration** since **air resistance opposes its motion.** As such, the ball will <u>**reach a shorter maximum horizontal distance.**</u>



Changes to path:
- Shorter maximum height reached
- Shorter maximum distance
- No longer **parabolic.**