MARCH PRACTICE QUESTIONS 2021
SECONDARY 4 EXPRESS
SECONDARY 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS   **4047/01**

Specimen Paper

Date: **3 March 2021**   **Duration: NIL**

Candidates answer on separate writing paper

READ THESE INSTRUCTIONS FIRST

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For π, use either your calculator value of 3.142, unless the question requires the answer in terms of π.

Topic names will be listed above each question for your benefit and revision

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Upon completion of solutions:

Each candidate have exactly 2 weeks to submit their solutions
Take a picture or send the digital version of your solutions to me (Kaiwen) via Telegram (**@kaiwen_tutor**) or WhatsApp (**90583779**)
Ensure that all workings are clear and legible
Solutions will be marked based on your presentation, accuracy and completeness of your solutions
A markers' report and the full solutions will be provided at the end of the month

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Setter: Ong Kai Wen

This question paper consists of <u>9</u> printed pages including the cover page

Content Covered
- Quadratic Functions, Equations & Inequalities
- Indices & Surds
- Polynomials
- Partial Fractions
- Binomial Theorem
- Power, Exponential, Logarithm & Modulus Functions
- Trigonometry
- Linear Law
- Coordinate Geometry
- Further Coordinate Geometry
- Proofs of Plane Geometry
- Calculus

All questions are sourced and selected based on the known abilities of students sitting for the 'O' Level A-Math Examination. If questions are sourced from respective sources, credit will be given when appropriate

Special Note from Tutor (Kaiwen):

Some of these questions are slightly more challenging than others and require some out of the box thinking. When faced with such challenging questions, always go back to the fundamentals and think about the basics you already have learnt in school. Questions will never deviate away from the curriculum that is already pre-set for you

Nonetheless, don't give up if you are unable to solve the questions! Send in your solutions as how you would submit your answer scripts during the National Examinations. From there, I will be able to see and judge the ability of the cohort before moving on and planning the curriculum and content for the rest of the year.

All the best and I really do hope that this initiative will help as many students as it can reach! 加油!

The roots of the quadratic equation $x^2 - 2(m-1)x + (m^2 - 7) = 0$ are α and β. Given that $\alpha^2 + \beta^2 = 10$, find the value of m and obtain an equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

Given that a and b are roots of the equation $x^2 + x - 1 = 0$ and $a < b$, prove that

$$\frac{1}{\sqrt{3}} a^2 b = \frac{\sqrt{5} + 1}{2\sqrt{3}}$$

Show that $my = x^2 - 4(x-1)$ meets the curve $y = x^2 - 3x + 2$ at 2 distinct points for all non-zero values of m

Solve the simultaneous equations

$$9^y \times 27 = 3^{2x-1}$$
$$7^y \sqrt{7^x} = 343$$

Topic: Indices & Surds
Without using a calculator, find the value of a and b for which

$$\frac{6}{\sqrt{2}}\left(\frac{5\sqrt{32}}{2}+\frac{15}{\sqrt{50}}-\frac{14}{7\sqrt{6}}\right)=a-b\sqrt{3}$$

Topic: Polynomials
Given that $4x^4-px^3-9x^2+x+2=(x-2)(x+1)(ax^2-b)$, find a, b and p

Topic: Partial Fractions
Express the following in partial fractions:

$$\frac{123x-46}{(2-x)(3x-1)^2}$$

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Topic: Binomial Theorem

In the given expansion, where $n > 0$, the first **3** terms in ascending powers of x are

$$\left(\frac{k}{x^3} + cx\right)^n = \frac{1}{x^{66}} + \frac{44}{x^{62}} + \frac{924}{x^{58}} + \cdots$$

(a) Find the values of k, n and c

(b) In the given expansion, where p is a positive constant, the term independent of x is **5376**. Find the value of p

$$\left(x^2 - \frac{p}{2x}\right)^9$$

[S4 CCHY P2/2012 PRELIM Qn 2]

Topic: Power, Exponential, Logarithm & Modulus Functions

Solve the following equation
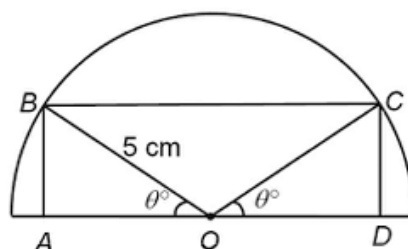
$$\log_9 3x - \log_3 3x = \log_{\sqrt{3}} 3$$

Solve the equation

$$\log_4 (3x - 1)^2 - \frac{2}{\log_{\sqrt{2}} 2} = \log_4 \left(x^2 + \frac{9}{4}\right)$$

Prove that

$$\frac{\sin A + \sin 2A + \sin 3A}{\cos A + \cos 2A + \cos 3A} = \tan 2A$$

The diagram on the right shows a rectangle $ABCD$ inside a semicircle, centre O and radius $5cm$, such that $\angle BOA = \angle COD = \theta°$

(a) Show that the perimeter, $P\ cm$, of the rectangle is given by the formula $P = 20\cos\theta + 10\sin\theta$

(b) Express P in the form $R\cos(\theta - \alpha)$ and hence find the value of θ for which $P = 16$

(c) Find the value of k for which the area of the rectangle is $k\sin 2\theta\ cm^2$

Prove that, for all real values of x

$$\frac{\cot x}{\sqrt{1 + \cot^2 x}} - \frac{\cosec x}{\tan x + \cot x} = 0$$

Solve for y, between $0°$ and $360°$ for the equation

$$4\cosec^2 y = 7 - \cot^2 y + 2\cot y$$

Topic: Linear Law

Variables x and y are related by the equation $y = a\sqrt{x} + \frac{b}{\sqrt{x}}$, where a and b are constants. The table below shows measured values of x and y

x	1	2	3	4	5	6
y	5.7	5.6	5.9	6.2	6.6	6.9

(a) On a piece of graph paper, plot $y\sqrt{x}$ against $x,$ using a scale of $2\ cm$ to represent 1 unit on the $y\sqrt{x}$ axis. Draw a straight line graph to represent the equation $y = a\sqrt{x} + \frac{b}{\sqrt{x}}$

(b) Use your graph to estimate the value of a and of b

(c) On the same diagram, draw the line representing the equation $y = \frac{3x}{\sqrt{x}}$ and hence find the value for which

$$x = \frac{b}{3 - a}$$

Topic: Coordinate Geometry
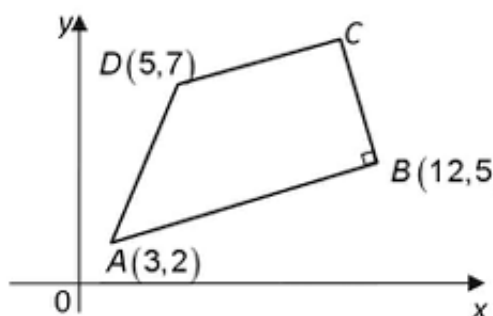
**Solutions to this question by accurate drawing will not be accepted**

In the diagram on the right, AB is parallel to DC and $\angle ABC = 90°$. Given that the coordinates of A, B and D are $(3, 2)$, $(12, 5)$ and $(5, 7)$ respectively, find

(a) The equations of BC and DC

(b) The coordinates of C

(c) The equation of the perpendicular bisector of AB

Topic: Further Coordinate Geometry

The straight line $3x - y + 5 = 0$ and the curve $x^2 + y^2 - 2x - 6y + 5 = 0$ intersect 2 points A and B. Find the coordinates of A and B. Hence, find the length of AB

Topic: Proofs of Plane Geometry
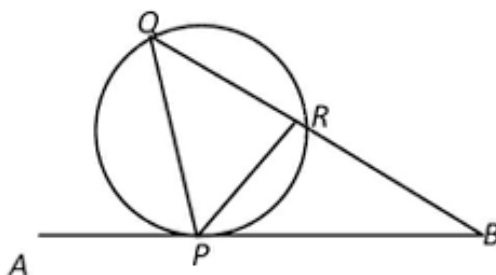
The diagram on the right shows a triangle PQR inscribed in a circle and AB is a tangent to the circle at P. PR bisects $\angle QPB$. Prove that
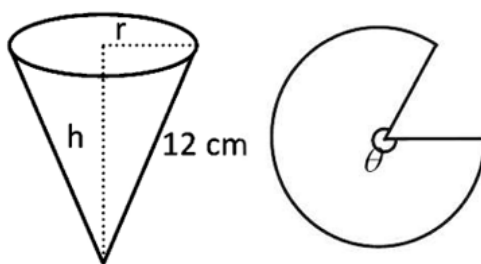 (a) $RQ = RP$
  (b) $BR \times RP = BP^2 - BR^2$

Topic: Calculus

Show that

$$\frac{d}{dx}\left(\frac{x}{1+5x}\right) = \frac{1}{(1+5x)^2}$$

Hence, or otherwise, find the area bounded by the x-axis, the lines $x = 1$, the line $x = 3$, and the curve

$$y = \left(\frac{4}{1+5x}\right)^2$$

Peter wishes to make a cone to hold water. He makes a right circular cone, without overlap, of depth h cm, radius r cm and slant height 12 cm

(a) Show that the volume of the cone, V cm^3 is given by the equation

$$V = 48\pi h - \frac{1}{3}\pi h^3$$

(b) Find the value of h for which V has a stationary value. Find this value of V and determine whether it is a maximum or minimum

(c) Given that Peter uses material that is originally in the form of a sector of angle θ radians. Show that θ is approximately 5.13 when V is stationary

Evaluate the following

$$\int \frac{2x^2 + 16x}{(1-3x)(2x+1)^2}\, dx$$