Topic 14: Current of Electricity

Key Concepts

- ❑Electric Current
- ❑Potential Difference (p.d.)
- ❑Resistance and Resistivity
- ❑Source of Electromotive Force (emf)

Electric Current

- the net flow of charge through a material
- The rate at which charge flows through a surface

- Symbol: I
- SI unit: Ampere (A)
- Scalar

$$I = \frac{dQ}{dt}$$

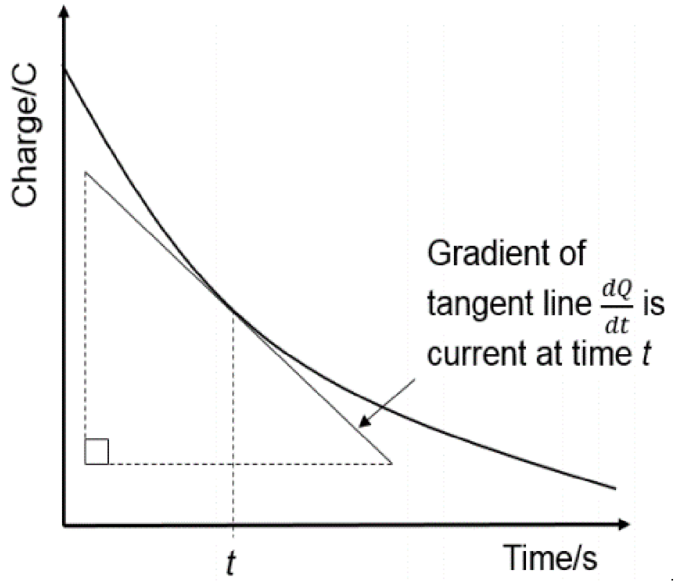
- Instantaneous current

$$I_{ave} = \frac{\Delta Q}{\Delta t}$$

- Average current

- where charge Q is in coulomb (C) and time t is in seconds (s)

Graph



- The conventional direction of current = the direction of **flow of positive charges.**
- For negatively charges, the current direction is opposite to the conventional current.
- In a circuit, the conventional current flows from positive (+) to negative (-) terminal.
- Conversely, the electron flows from (-) to (+) terminal.

Current and Drift Velocity

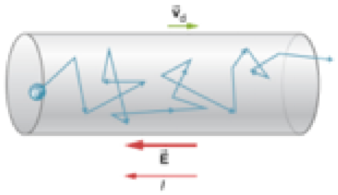
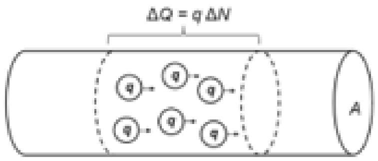
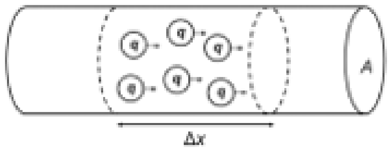
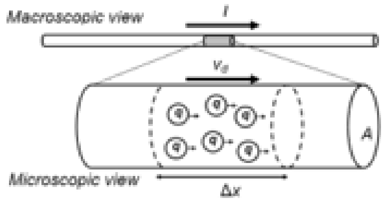
$$\boxed{I = nAv_d q}$$

n = number density of charge carriers (number of charge carriers per unit volume ($1/m^3$)
A = cross-sectional area of the conductor (m^2)
v_d = drift velocity (m/s)
q = charge of each carrier (C)

Number density

$$\boxed{n = \frac{N}{V} = \frac{N_A \rho}{M}}$$

where N is the total number of objects (charge carriers) in volume V.
$N_A = 6.02 \times 10^{23}\ mol^{-1}$ (Avogadro's number),
ρ = mass density (kg/m^3)
M = molar mass (kg/mol)

Derivation	
	• The potential difference applied across a conductor sets up electric field $\vec{E}$ in the conductor. • Free electrons feel a force due to $\vec{E}$ which results to a motion opposite to $\vec{E}$. • Since electrons are negatively charged, the direction of current I is opposite to their motion. • Electrons collide with other electrons and atoms in the conductor as shown in the left figure.
	• Let ΔN= number of charge carriers in the segment. • q = charge of each carrier • The total charge ΔQ in the segment is simply $$\Delta Q = q\Delta N$$
	• The number density n of charge carriers is given by $$n = \frac{\Delta N}{\Delta V} \rightarrow \Delta N = n\Delta V$$ • where the volume ΔV of the segment is just the volume of a cylinder. • Given by the cross-sectional area A and length Δx of the segment $$\Delta V = A\Delta x$$
	• Take note that $I = \Delta Q/\Delta t$ $$I = \frac{q(n\Delta V)}{\Delta t} = \frac{nqA\Delta x}{\Delta t}$$ • Drift velocity is simply the displacement over time $$v_d = \Delta x/\Delta t$$ • Finally, $$I = nAv_d q$$

Electric Charge

- The charge which flows past a point time t is the product of current and time

- Symbol: Q
- SI unit: Coulomb (C)
- Scalar

$$\boxed{Q = \int I dt}$$

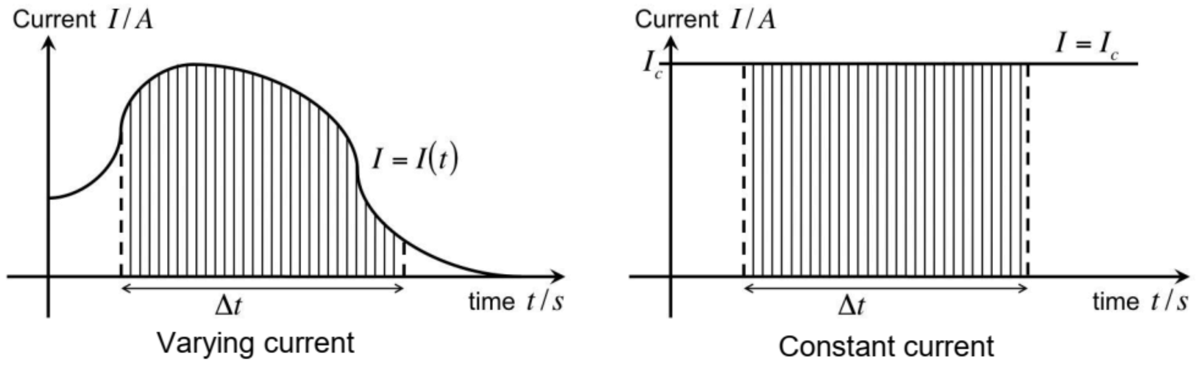
- In general

$$\boxed{Q = It}$$

- for constant current

- where charge I is in ampere (A) and time t is in seconds (s)

Graph



- Q is the area under the curve (I vs t)

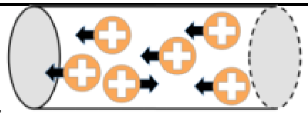
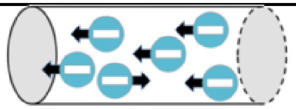
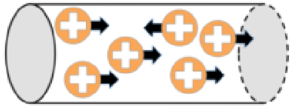
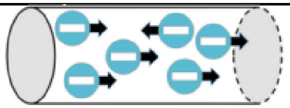
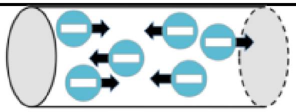
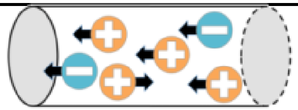
- charge of proton: $1.60 \times 10^{-19} C = q_e$
- charge of electron: $-q_e$

charge is quantized, meaning that any charge can only exist as integer multiple of q_e. That is $Q = nq_e$ where n is an integer.

Practice Example 1

Practice Example 1

Indicate the conventional direction of current for the following cases. Write *left*, *right*, or *no current*.

	Current direction		Current direction
a. 	left	d. 	right
b. 	right	e. 	left
c. 	No current	f. 	No current

Practice Example 2

A charging station in a convenience store charges a cellphone battery using a current of $1.5\ A$ for 3.0 hours. How much charge passed though the battery?

Note that unit used for the time is in hours. So we have to convert it in seconds. There are 60 seconds in 1 minute and there are 60 minutes in 1 hr.

$$Q = It$$

$$Q = (1.5A)(3.0hrs)\left(\frac{60min}{1hr}\right)\left(\frac{60s}{1min}\right)$$

$$Q = 16.2 \times 10^3 C$$

Practice Example 3

Calculate the drift velocity of electrons in an aluminum wire carrying a $10.0-A$ current. The wire has cross-sectional area equal to $3.5\times10^{-6}m^2$. Assume that each aluminum atom supplies one free electron per atom.

Density of aluminum: $\rho_{Al}=2.70g/cm^3$
Molar mass of aluminum: $M_{Al}=26.98\ g/mol$
Electron charge: $q=1.60\times10^{-19}C$
Avogadro's number: $N_A=6.02\times10^{23}\ mol^{-1}$

$$V=\frac{M}{\rho}\rightarrow n=\frac{N_A\rho}{M}$$

Calculating for the drift velocity,

$$v_d=\frac{I}{nqA}=\frac{IM_{Al}}{qAN_A\rho_{Al}}=\frac{(10A)(26.98\ g/mol)}{(1.6\times10^{-19}C)(3.5\times10^{-6}m^2)(6.02\times10^{23}\ mol^{-1})(2.70g/cm^3)}$$

Converting $g\rightarrow kg$ and $cm^3\rightarrow m^3$

$$v_d=\frac{(10A)(0.0027\ kg/mol)}{(1.6\times10^{-19}C)(3.5\times10^{-6}m^2)(6.02\times10^{23}\ mol^{-1})(2{,}700kg/m^3)}$$

$$v_d=2.97\times10^{-5}m/s$$

Important: always convert values in their corresponding SI units.

Electric Potential

- Symbol: *V*
- SI unit: Coulomb (C)
- Scalar

- Conventional current flows from a point of higher (+ terminal) to lower (-terminal) potential.
- is a characteristic of the electric field only.
- does not depend on test charge just like electric field.
- is not measurable (not physically meaningful as it varies depending on the chosen reference point).

- Difference in potential is the quantity that is physically meaningful

Potential Difference and emf

POTENTIAL DIFFERENCE, p.d. (across two points)	**ELECTROMOTIVE FORCE, *emf*** (of a source)
Symbol: V SI unit: Volts, V Scalar	Symbol: ε SI unit: Volts, V Scalar
is the electrical energy **converted to** other forms of energy, per unit charge passing through the device.	is the electrical energy **converted to** other forms of energy, per unit charge passing through the device
$V = \frac{W}{Q}$ W = amount of electric potential energy **converted to** other forms of energy	$\varepsilon = \frac{W}{Q}$ W = amount of electric potential energy **converted from** other forms of energy
• Equal to zero when there is no current in the circuit ($IR = 0$) • Depends on the equivalent resistance of the circuit/device.	• Maximum possible voltage the battery (or any voltage source) can provide between its terminals • Independent of I or R
	Misconceptions: • ε is not a force • a battery does not provide constant current in a circuit. The battery is a source of constant emf ε.

Electric Power

- The charge which flows past a point time t is the product of current and time

- Symbol: P
- SI unit: Watt (W) = Joule/second (J/s)
- Scalar

$$P = \frac{dW}{dt} = IV = I^2R = \frac{V^2}{R}$$

- In general

$$P = \frac{E}{t}$$

- If power is delivered at constant rate

W (or E) $=$ the amount of energy converted from electric potential energy to other forms of energy.
V $=$ p.d across the device (or e.m.f.)
I $=$ current passing through the device
R $=$ resistance of the device/circuit

Practice Example 4

An average cloud-to-ground lightning bolt contains roughly $10^9 J$ across a potential difference of 1.3×10^8 V. This event typically lasts for about 0.25 s

(a) Estimate the total amount of charge that is transferred from cloud to the ground?

Using $V = \frac{W}{Q}$,

$$Q = \frac{W}{V} = \frac{10^9 J}{1.3 \times 10^8 V} = 7.69\ C$$

(b) Calculate the current during the 0.25s duration.

$$I = \frac{Q}{t} = \frac{7.69C}{0.25s} = 30.76A$$

Practice Example 5

One of the fastest chargers in 2020 delivers 65W of power to most flagship phones. Suppose that the charger's output voltage is $5.0V$, it will take about 30 minutes to fully charge a phone.

(a) Calculate the total energy converted by the charger from electrical energy?

$$E = Pt = (65W)(30min)\left(\frac{60s}{1min}\right)$$

$$= 117 \times 10^3 = 117kJ$$

(b) Determine how much current flows through the charger.

$$I = \frac{P}{V} = \frac{65W}{5V} = 13A$$

(c) Calculate the total charge flowed from the charger.

$$Q = It = (13A)(30min)\left(\frac{60s}{1min}\right) = 23.4 \times 10^3 C$$

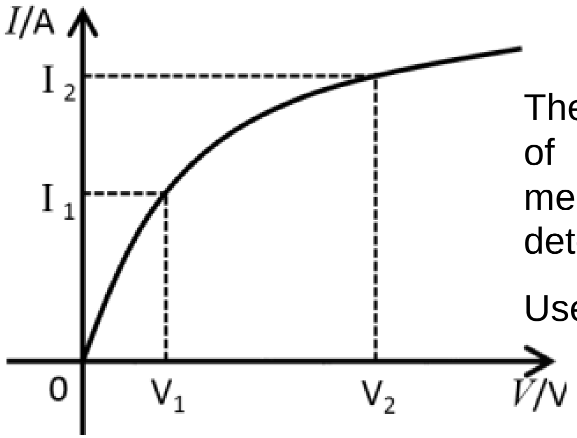
Resistance

- ratio of the potential difference across a conducting material to the current flowing thought it.
- is not constant for a given material but depends on the shape, size, and temperature.

- Symbol: R
- SI unit: Ohm (Ω)
- Scalar

$$R = \frac{V}{I}$$

- V = potential diff. across the conductor
- I = current through the conductor



The slope of the tangent line or the gradient of *I-V* graph does not carry any physical meaning and thus cannot be used to determine the resistance of the conductor

Use the ratio of V to I for the resistance.

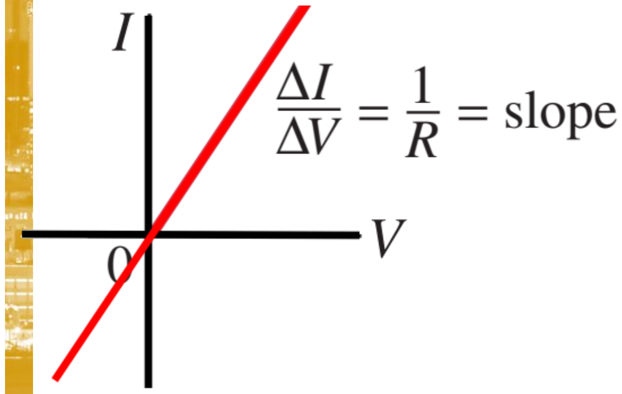
Ohm's Law

- the **current** through a conductor between two points **is proportional** to the **potential difference** across the two points.

*under constant physical conditions (e.g. temperature, mechanical stress)

$$V \propto I$$

$$V = IR$$



Features:

- linear (straight)
- passes through origin

- The resistance $R = V/I$ is constant because the current increases with increasing p.d.

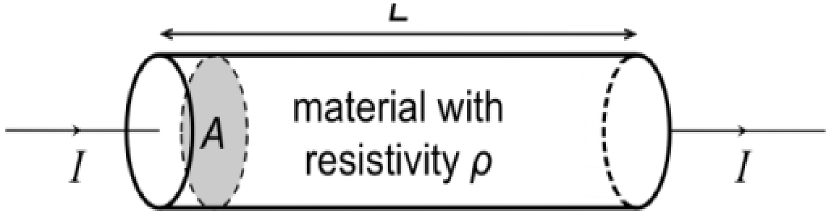
A material that follows Ohm's law is called **ohmic**.

Resistivity

- Is a measure of how strongly a material opposes electric current.
  - It characterizes the resistance of materials at a fixed temperature regardless of the materials’ dimension (shape and size).
  - Its values is constant for a specific material, unlike resistance.

$$R = \rho \frac{L}{A}$$

ρ = resistivity of the conductor
L = length
A = cross-sectional area



Key pointers

Resistance to current

- directly proportional to the length of the conductor
- inversely proportional to the cross-sectional area

ρ is proportionality constant that relates R to the dimension of the conductor

Practice Example 6

The diameter of a Nichrome wire that is used as a heating element in an electronic device is equal to $5.00\ mm$. In order to draw 8.50-A current when the potential difference across its ends is $120\ V$, what should be the

(Resistivity of Nichrome: $\rho = 1.00 \times 10^{-6}\Omega \cdot m$)

(a) required resistance of the wire?

$$R = \frac{V}{I} = \frac{120V}{8.50A} = 14.12\Omega$$

(b) length of wire you must use?

Let d= diameter of the wire such that

$$A = \pi\left(\frac{d}{2}\right)^2 = 3.14\left(\frac{5.00\ \times 10^{-3}m}{2}\right)^2$$

$$= 1.96 \times 10^{-5}m^2$$

$$L = \frac{AR}{\rho} = \frac{(1.96 \times 10^{-5}m^2)(14.12\Omega)}{1.00 \times 10^{-6}\Omega \cdot m} = 277.25m$$

Practice Example 7

The resistance of an aluminum wire is the twice the resistance of a copper wire. If the two wires have the same length, what is the ratio $\frac{r_{Al}}{r_{Cu}}$ of their radii?

Resistivity of aluminum: $\rho_{Al} = 2.82 \times 10^{-8}\Omega \cdot m$
Resistivity of copper: $\rho_{Cu} = 1.70 \times 10^{-8}\Omega \cdot m$

$$A = \rho\frac{L}{R}$$

$$\pi r^2 = \frac{\rho L}{R} \rightarrow r = \sqrt{\frac{\rho L}{\pi R}}$$

Note that $R_{Al} = 2R_{Cu}$ and $L_{Al} = L_{Cu} = L$

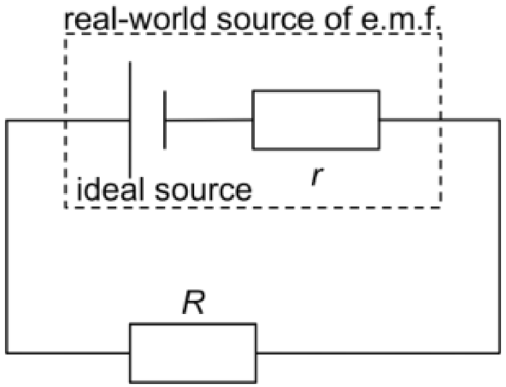
$$\frac{r_{Al}}{r_{Cu}} = \frac{\sqrt{\frac{\rho_{Al}L}{\pi 2R_{Cu}}}}{\sqrt{\frac{\rho_{Cu}L}{\pi R_{Cu}}}} = \sqrt{\frac{\rho_{Al}}{2\rho_{Cu}}}$$

$$\frac{r_{Al}}{r_{Cu}} = \sqrt{\frac{2.82 \times 10^{-8}\Omega \cdot m}{2(1.70 \times 10^{-8}\Omega \cdot m)}} = 0.91$$

Internal Resistance

- resistance offered by an emf source

- Symbol: r
- SI unit: Ohm (Ω)
- Scalar



$$\varepsilon = I(R + r)$$

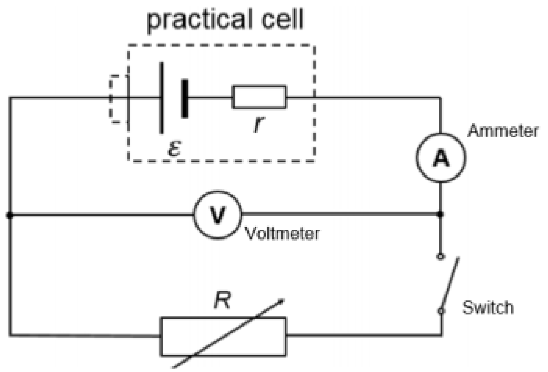
I =current flowing in the circuit
R = resistance of the external load R
r = internal resistance

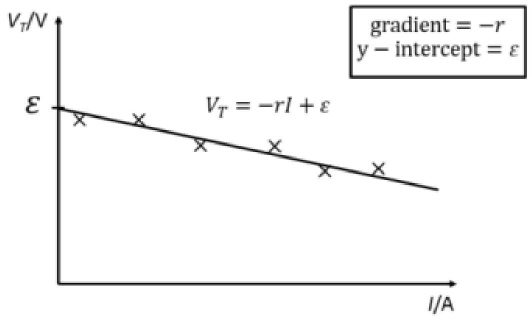
$$P = I\varepsilon = I^2(R + r)$$

- Total power output of emf source

$$V_T = IR = \varepsilon - Ir$$

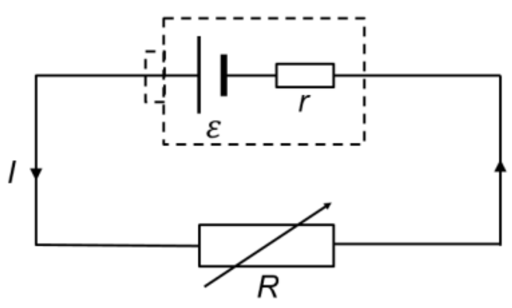
- Terminal voltage

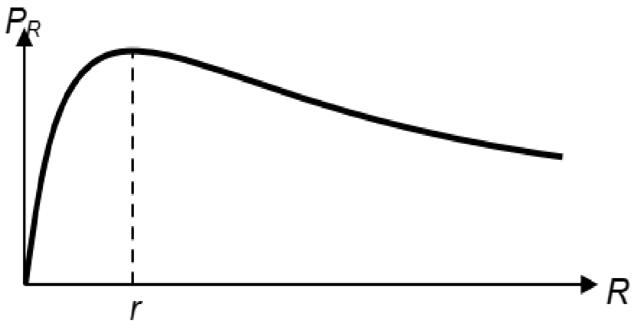
MEASURING INTERNAL RESISTANCE	
Circuit Diagram	
	When switch is **open**, • the voltmeter measures the **emf** of the source. When the switch is **closed**, • the voltmeter reads the **terminal voltage** V_T of the source • the ammeter I measures the **current** in the circuit

Graph (Measuring internal resistance)	
	• The V_T vs. I graph will help us determine r • Slope of the graph gives r • y-intercept gives the emf ε

Maximum Power Theorem

- The resistance of the load must be the same as the internal resistance $(R = r)$, for the **power** transferred from the e.m.f. source to the external load to be **maximum**





Variation of power P_R dissipated in R with resistance R

Assumptions:

- constant power supplied by the source
- Energy is only dissipated at the external load R and internal resistance r

Derivation

The power delivered to the external load R is given by

$$P_R = I^2 R$$

The circuit is the same as the circuit in the discussion box for internal resistance so we can use the emf formula to obtain equation for current I

$$I = \frac{\varepsilon}{R + r}$$

This gives us

$$P_R = \left(\frac{\varepsilon}{R + r}\right)^2 R = \varepsilon^2 \frac{R}{(R + r)^2}$$

At the maximum R, the slope of the tangent line (which is a horizontal line) of the P_R vs R graph is zero. This means that to determine the value of R such that P_R is maximum, we have to solve for the gradient $\frac{dP_R}{dR} = 0$.

$$\frac{dP_R}{dR} = 0$$

$$\frac{d}{dR}\left[\varepsilon^2 \frac{R}{(R + r)^2}\right] = 0$$

$$\varepsilon^2 \frac{d}{dR}\left[R(R + r)^{-2}\right] = 0$$

Using product rule and cancelling ε^2

$$-2R(R + r)^{-3} + (R + r)^{-2} = 0$$

$$2R(R + r)^{-3} = (R + r)^{-2}$$

Getting the inverse and cancelling common term

$$\frac{R + r}{2R} = 1$$

$$2R = R + r$$

$$R = r$$

Practice Example 8

A battery has an internal resistance of 0.01Ω and emf of $12.3V$. Given that a load resistance of 5.0Ω is connected to its terminals,

(a) find the current in the circuit.

$$I = \frac{\varepsilon}{R + r} = \frac{12.3V}{(5.0 + 0.01)\Omega} = 2.46A$$

(b) Calculate terminal voltage of the battery

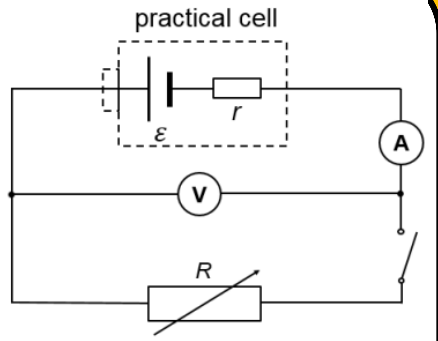
$$V_T = \varepsilon - Ir$$
$$V_T = 12.3V - (2.46A)(0.01\Omega) = 12.28V$$

(c) Find the power delivered to the load resistor, to the internal resistance of the battery, and by the battery.

To the load resistor: $P_R = I^2R = (2.46A)^2(5.0\Omega) = 30.26W$
To the internal resistance: $P_r = I^2r = (2.46A)^2(0.01\Omega) = 0.61W$
By the battery: $P = P_R + P_r = 30.26W + 0.61W = 30.87W$

Practice Example 9

Shown on the left is the circuit diagram consisting of a battery and a variable resistor R. The switch was turned on and the voltmeter measured $8.0V$ when the resistance was 20.0Ω. As the resistance is increased to 28.33Ω, the voltage became $8.5V$. What is the internal resistance r?



Note that the voltmeter reads the terminal voltage V_T of the batter.
For the first reading:

$$I = \frac{V_T}{R} = \frac{8.0V}{20.0\Omega} = 0.40A$$

The corresponding equation for the emf is

$$\varepsilon = V_T + Ir = 8.0V + (0.40\text{A})r$$

For the 2nd reading:

$$I = \frac{V_T}{R} = \frac{8.5V}{28.33\Omega} = 0.30A$$

Which corresponds to

$$\varepsilon = 8.5V + (03A)r$$

We have two equation with two unknowns. Subtracting the 2nd equation from the 1st equation of emf:

$$0 = (8.0 - 8.5)V + (0.4A - 0.30A)r$$
$$r = \frac{0.5V}{0.1A} = 5.0\Omega$$